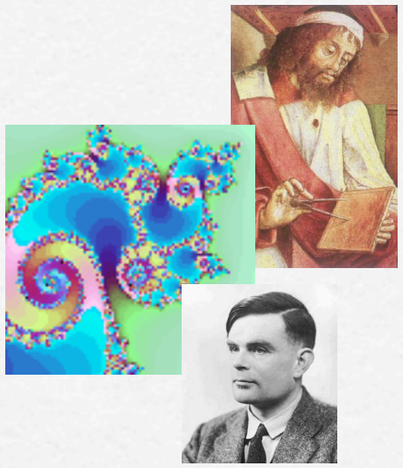
How Can Nature Help Us Compute?

a computability theoretic overview



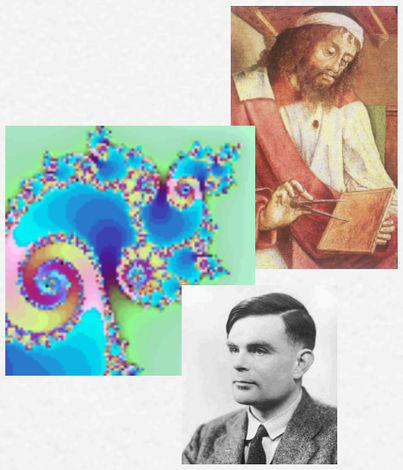
The relativity of "Horizons"

- Mathematical horizons are more schematic and reductive

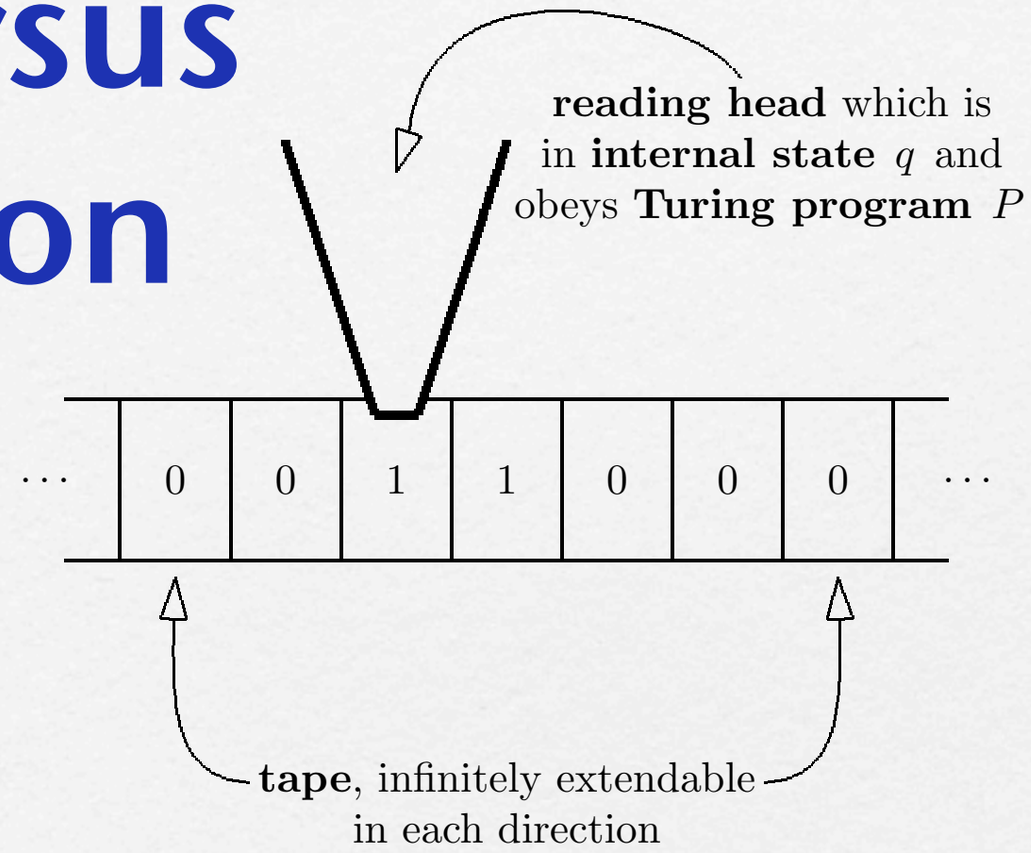
- They deal with underlying principles, and hence . . . more universally operative

- They may stretch further into the distance - but are less obviously relevant

- The mathematics may appear to originate and have significance independently of the real world - an illusion!



Process versus Computation



- Turing's "universal" model of computation
- Emergence in Nature algorithmic? Modelling chaotic, or quantum phenomena?
- Growth of new computational paradigms based on metaphors for natural phenomena, and of informative computer simulations got from copying nature

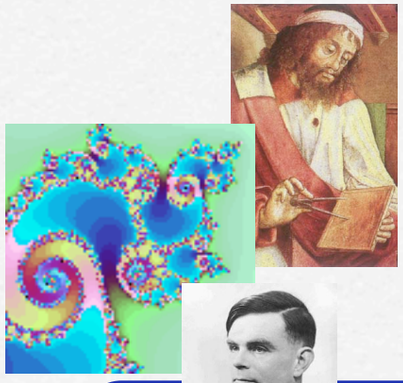
QUESTIONS . . .

- Can natural computation, in its various forms, provide us with genuinely new ways of computing?

- To what extent can natural processes be captured computationally?

- Is there a universal model underlying these new paradigms?

- Or is natural computation essentially an ad hoc activity? Theoretical limitations?

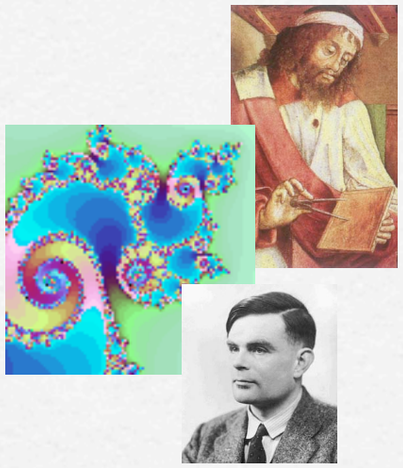


Observation, Prediction and Theory

"According to Bacon, scientists should travel over the earth collecting facts, until the accumulated facts reveal how Nature works. The scientists will then induce from the facts the laws that Nature obeys.

According to Descartes, scientists should stay at home and deduce the laws of Nature by pure thought. ... Faraday and Darwin and Rutherford were Baconians: Pascal and Laplace and Poincaré were Cartesians. Science was greatly enriched by the cross-fertilization of the two contrasting ... cultures."

from Freeman Dyson's introduction to George Odifreddi's "The Mathematical Century : The 30 Greatest Problems of the Last 100 Years"

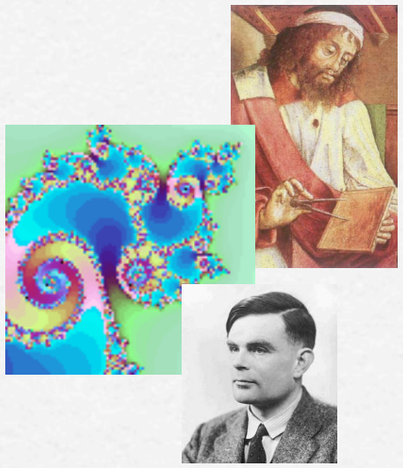


The Algorithmic Content of Science

- Newton onwards - overarching aim of science became the extraction of the algorithmic content of the world

- Einstein [p.54, `Out of My Later Years', 1950]: "When we say that we understand a group of natural phenomena, we mean that we have found a constructive theory which embraces them."

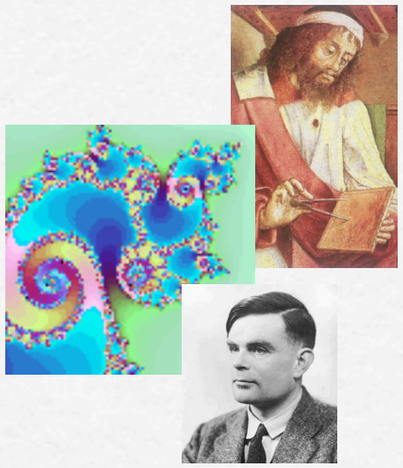
- With the constraint of Popperian falsification as reinforcement



Laplace's Predictive 'Demon'

"Given for one instant an intelligence which could comprehend all the forces by which nature is animated and the respective situations of the beings who compose it - an intelligence sufficiently vast to submit these data to analysis - it would embrace in the same formula the movements of the greatest bodies and those of the lightest atom; for it, nothing would be uncertain and the future, as the past, would be present to its eyes."

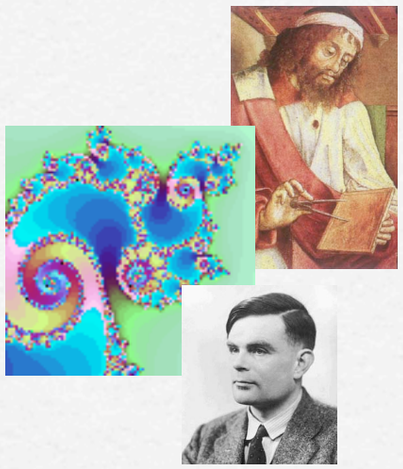
from P. S. de Laplace [1819], "Essai philosophique sur les probabilités"



Hilbert's Programme

"For the mathematician there is no Ignorabimus, and, in my opinion, not at all for natural science either. ... The true reason why [no one] has succeeded in finding an unsolvable problem is, in my opinion, that there is no unsolvable problem.
In contrast to the foolish Ignorabimus, our credo avers:
We must know,
We shall know. "

- David Hilbert's opening address to the Society of German Scientists and Physicians, Königsberg, September 1930

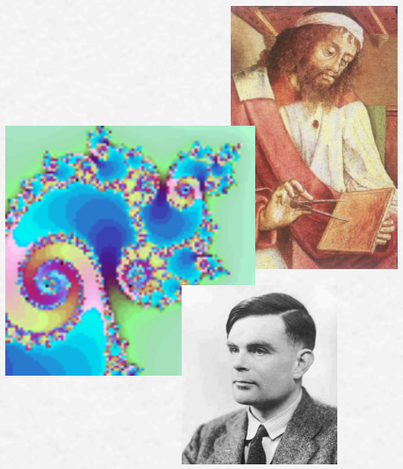


Unpredictability in mathematics

- Since 1936 we have a good idea of what an algorithm is (Church-Turing Thesis) - can talk about computable real numbers

- Take a computable real number - say π - then ask: "Is there a sequence of exactly n 7's in the decimal expansion of π?"

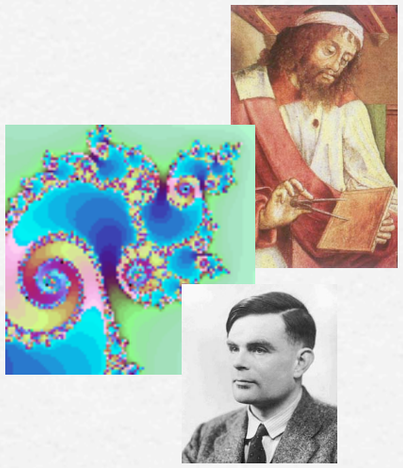
- May one day prove a theorem giving an answer for any given natural number n - but there are computable reals for which we know there can be no predictive program



Where are the uncompleted infinities?

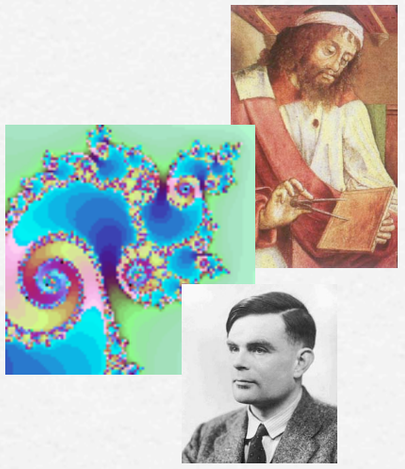
"Shine a bright light on the graph of a computable function over the natural numbers, and its shadow is likely to be incomputable - or more precisely, the projection of a suitable computable binary relation over the numbers produces an incomputable set of numbers.
So it seems we already knew that you get incomputable objects by selectively observing algorithmic processes.
The only problem with our ... example is one seems to need infinitely much time, which makes the computable simulation of incomputability interesting, but difficult to connect with our own world."

SBC, Definability as hypercomputational effect, Applied Mathematics and Computation, to appear.



Incomputability into the long grass . . .

- The birth of Recursion Theory, and an emphasis on purely mathematical issues (extending to logic in general)
- The growing belief that mathematics - and science in general - could carry on much as before. Natural examples?
- Discovery (J. Myhill) that all the unsolvable problems discovered in the 1930s were basically the same
- Richness of the computable universe revealed

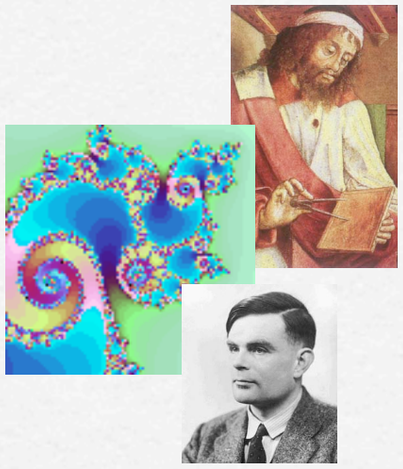


Natural phenomena as discipline problem

- Incomputability as a mathematical reality - e.g., inability to predict the halting of universal computing machines, or to tell if an argument is logically valid

- Reduction of "natural" examples to Turing model - e.g. quantum computation

- Martin Davis versus the hypercomputationalists (Jack Copeland et al)

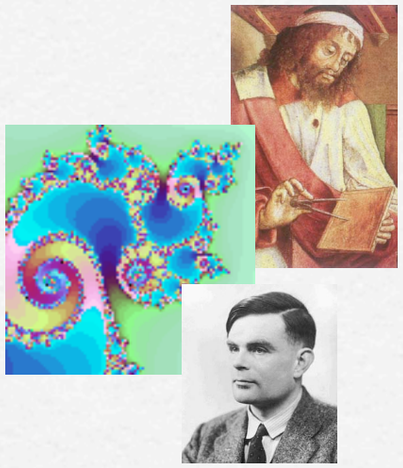


Natural phenomena as discipline problem

> "The great success of modern computers as all-purpose algorithm-executing engines embodying Turing's universal computer in physical form, makes it extremely plausible that the abstract theory of computability gives the correct answer to the question 'What is a computation?', and, by itself, makes the existence of any more general form of computation extremely doubtful."

Martin Davis [2004], The myth of hypercomputation. In Alan Turing: Life and legacy of a great thinker (C. Teuscher, ed.), Springer-Verlag

But - Philip Welch [2004] - logical proofs of the impossibility of hypercomputation "may be akin to proofs that fairies are logically impossible: damn hard to be convincing."



Natural phenomena as discipline problem

- Persistence of problems of predictability - at quantum level, relation between emergence and chaos, relativistic phenomena (see Istvan Németi and Hajnal Andreka, 2005)

- Renewed interest in analog and hybrid computing machines leading to: " ... the classical Turing paradigm may no longer be fully appropriate to capture all features of present-day computing."

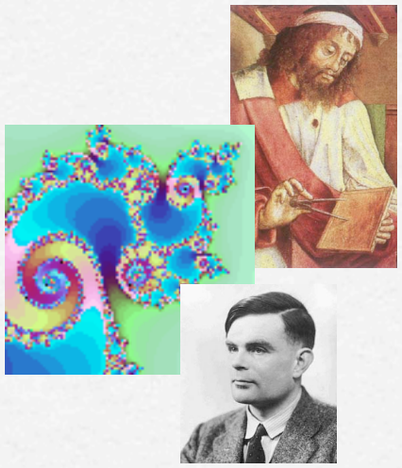
- J. van Leeuwen, J. Wiedermann, The Turing Machine Paradigm in Contemporary Computing. In Mathematics Unlimited - 2001 and Beyond, LNCS, 2000

"Von Neumann's axioms distinguished the **U** (unitary evolution) and **R** (reduction) rules of quantum mechanics. Now, quantum computing so far (in the work of Feynman, Deutsch, Shor, etc) is based on the **U** process and so computable. It has not made serious use of the **R** process: the unpredictable element that comes in with reduction, measurement, or collapse of the wave function."

Andrew Hodges

in "What would Alan Turing have done after 1954?", from Teuscher,

"Alan Turing: Life and legacy of a great thinker"

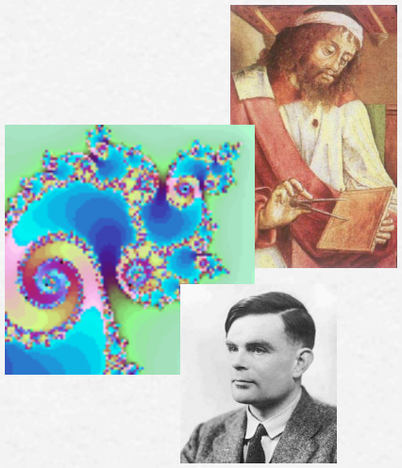


Co-operative phenomena

- 1970 - Georg Kreisel proposes a collision problem related to the 3-body problem, which might result in "an analog computation of a non-recursive function"

- Painlevé Problem (1897): Do non-collision singularities exist for the N-body problem for any N≥4?

- "Yes" - Jeff Xia, 1988, Saari and Xia, Off to Infinity in Finite Time, Not. Amer. Math. Soc. 42, 1995



Chaos and its Analogues

- Growth of Chaos theory, generation of informational complexity via very simple rules, accompanied by the emergence of new regularities - e.g. Robert Shaw [1984]

- Link between structures in nature, and mathematical objects, such as the Mandelbrot and Julia sets

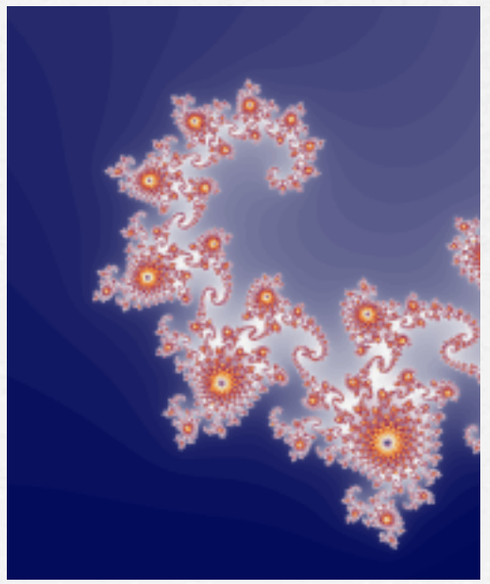
- Penrose, Smale - computability of Mandelbrot, Julia sets?

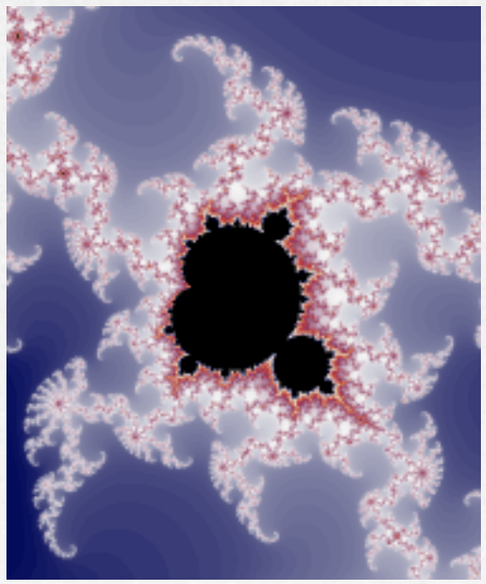
Now we witnessed ... a certain extraordinarily complicated looking set, namely the Mandelbrot set. Although the rules which provide its definition are surprisingly simple, the set itself exhibits an endless variety of highly elaborate structures.

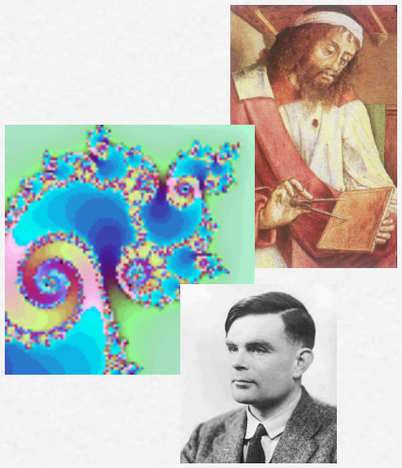
Roger Penrose

in "The Emperor's New mind", Oxford Univ. Press, 1994

- Recent results - Braverman [1999], Hertling [2005], Rettinger [2005], Rettinger and Weihrauch [2003]







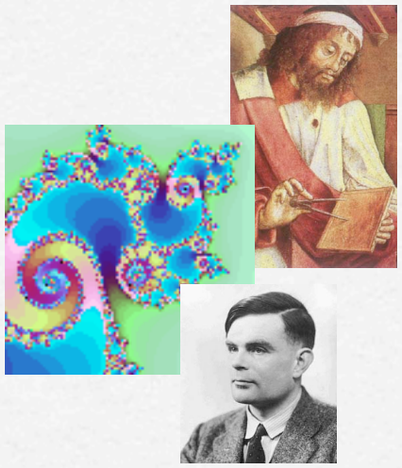
How far can the standard model stretch?

"One example of a problem that is not algorithmic is the following instruction from a recipe [quote Knuth, 1968]:

'toss lightly until the mixture is crumbly.'

This problem is not algorithmic because it is impossible for a computer to know how long to mix: this may depend on conditions such as humidity that cannot be predicted with certainty ahead of time.
In the function-based mathematical worldview, all inputs must be specified at the start of the computation, preventing the kind of feedback that would be necessary to determine when it's time to stop mixing."

D. Goldin, P. Wegner [2005], The Church-Turing Thesis: Breaking the Myth. In CiE 2005: New Computational Paradigms: Papers presented at the conference in Amsterdam, June 8-12, 2005, LNCS 3526

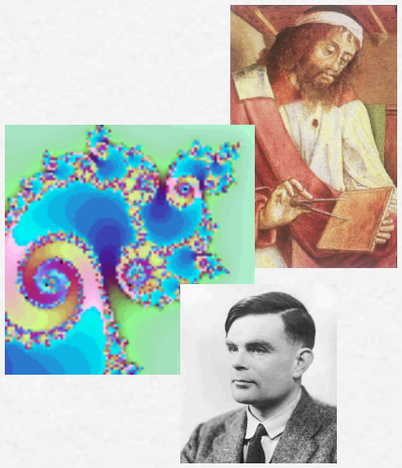


How far can the standard model stretch?

- Need to be wary of elevating important operational aspects into something more fundamental

- E.g. David Deutsch [The Fabric of Reality, 1997, p.210] - the massive parallelism delivered by quantum compution (as it is currently conceived) is perfectly well contained within the classical sequential model

- Question is - where is the phase transition with real paradigm testing potential?

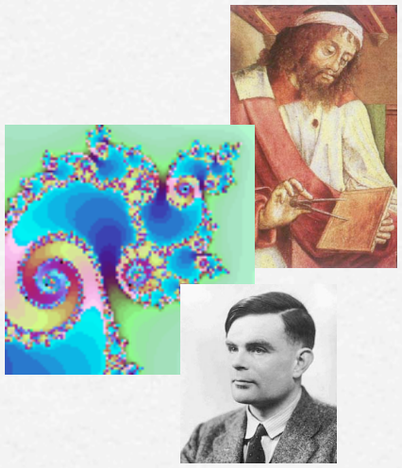


The elusive Global to Local connection

- Extreme example - The actuality of quantum non-locality, and lack of understanding of its connection with familiar physical laws (EPR, Bell's Inequality)

- Closer to home - Economic activity (e.g.) and its global constraints

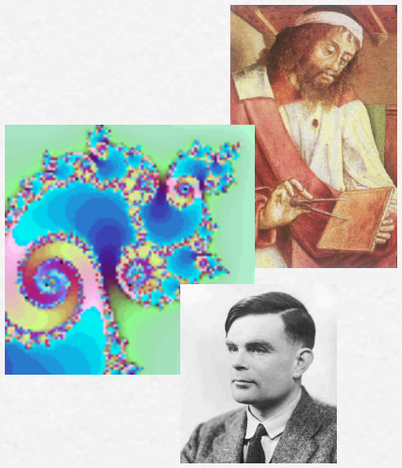
- Weather prediction, evolution, consciousness, of classical reality from quantum ambiguity, the origins of life, or of large structures in the Universe . . .



Descriptions and Emergent Structure

- Intuition - entities exist because of, and according to, mathematical laws. In the words of Leibniz [1714] -

- 'The Monadology', sections 31, 32: " ... there can be found no fact that is true or existent, or any true proposition, without there being a sufficient reason for its being so and not otherwise, although we cannot know these reasons in most cases."



Definability and Invariance

- So existence not only *generates* descriptions, but *arises* from them . . .

- Connecting with a useful abstraction – that of *mathematical definability* – or, more generally, *invariance* (under all automorphisms of a structure) . . .

- So providing a potentially *non-algorithmic determinant* of events

I believe the
following aspects of evolution to be true, without
knowing how to turn them into (respectable) research topics.

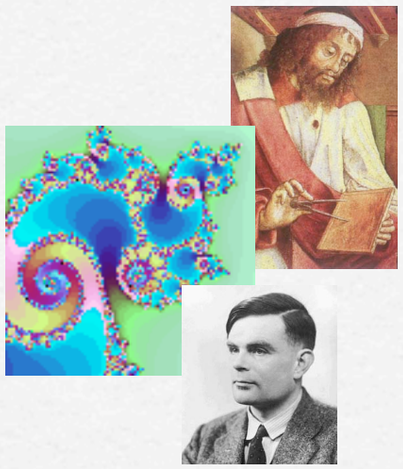
Important steps in evolution are robust. Multicellularity evolved at least ten times. There are several independent origins of eusociality. There were a number of lineages leading from primates to humans. If our ancestors had not evolved language, somebody else would have.

Cooperation and language define humanity. Every special trait of humans is a derivative of language.

Martin Nowak,

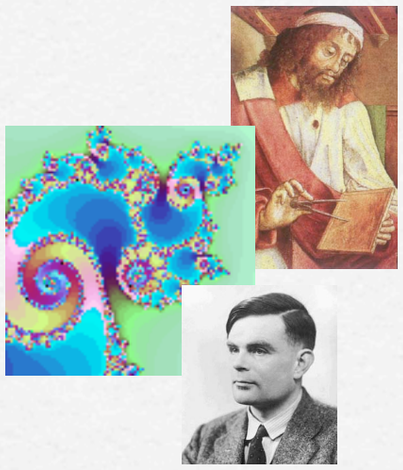
Director, Program for Evolutionary Dynamics, Harvard University,

in John Brockman (ed.): "What We Believe But Cannot Prove"



The Human Mind As Case Study

- Familiar - Baconian experience easily got through solving everyday problems, and observing others
- Mechanics of brain well-documented
- Does not feel, or appear to compute, like a Turing machine - role of creativity, consciousness, intuition
- Relevant - importance of Copying how humans think for AI etc . . .

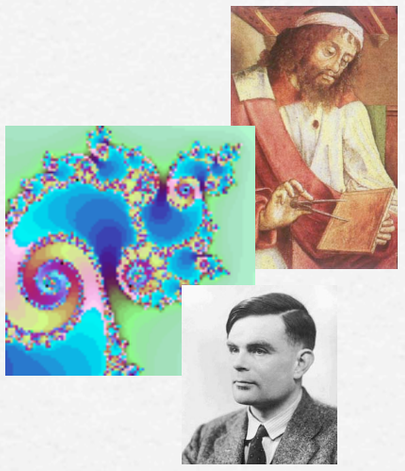


Hadamard on Mathematical Intuition

"At first Poincaré attacked [a problem] vainly for a fortnight, attempting to prove there could not be any such function ... [quoting Poincaré]:

'Having reached Coutances, we entered an omnibus to go some place or other. At the moment when I put my foot on the step, the idea came to me, without anything in my former thoughts seeming to have paved the way for it ... I did not verify the idea ... I went on with a conversation already commenced, but I felt a perfect certainty. On my return to Caen, for conscience sake, I verified the result at my leisure.' "

from Jacques Hadamard [1945], "The Psychology of Invention in the Mathematical Field", Princeton Univ. Press



Description versus Computation

- Turing, 1939 - The computational content of descriptions can be captured hierarchically - but in unpredictable ways

- No consistent theory captures arithmetic (Gödel)- but we can hierarchically transcend this barrier

- But then - identifying the route to a new theorem involves using an incomputable oracle

- Despite inductive structure, reductionism breaks down

Mathematical reasoning
may be regarded ... as the exercise of a combination of
... *intuition* and *ingenuity*. ... In pre-Gödel times it was thought by
some that all the intuitive judgements of mathematics could be
replaced by a finite number of ... rules. The necessity for intuition
would then be entirely eliminated.
In our discussions, however, we have gone to the opposite extreme and
eliminated not intuition but ingenuity, and this in spite of the
fact that our aim has been in much
the same direction.

Alan Turing [1939],
Systems of logic based on ordinals, Proc. London Math. Soc. (2) 45, pp.161-228.
Reprinted in A. M. Turing, Collected Works: Mathematical Logic, pp. 81-148.

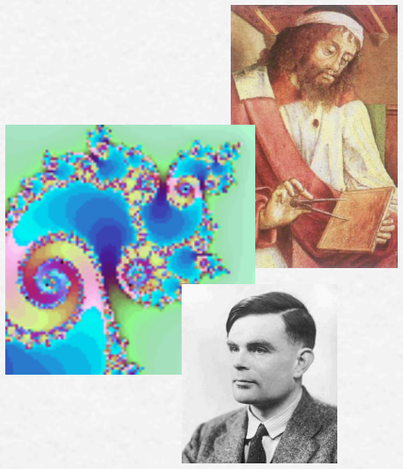
- An explanation of why written proofs do not tell us how the proof was discovered . . .

- And such ideas persisted: *"... if a machine is expected to be infallible, it cannot also be intelligent. There are several theorems which say almost exactly that."*

  - A.M. Turing, talk to the London Mathematical Society, February 20, 1947, quoted in Andrew Hodges, p.361

- "The results which have been described in this article are mainly of a negative character, setting certain bounds to what we can hope to achieve purely by reasoning. These, and some other results of mathematical logic may be regarded as going some way towards a demonstration, within mathematics itself, of the inadequacy of 'reason' unsupported by common sense."

  - final paragraph of Alan Turing, Solvable and Unsolvable Problems, Penguin Science News 31, 1954, p.23



Intelligent machines as emergent phenomena

- Need to bridge the gap between *higher mental functionality* and ... <u>what</u> *algorithmic context*?

<u>Difficult</u> - Rodney Brooks [Nature, 2001]: *"neither AI nor Alife has produced artifacts that could be confused with a living organism for more than an instant."*

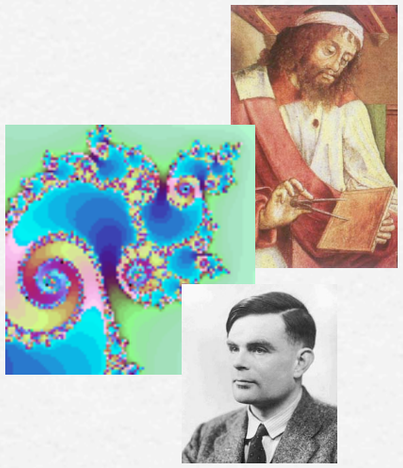
- But paradigm-stretching features in evidence in many different contexts . . .

There is a reasonable chance
that connectionist models will lead to the development of
new somewhat-general-purpose self-programming, massively parallel
analog computers, and a new theory of analog parallel computation: they
may possibly even challenge the strong construal of Church's Thesis as the
claim that the class of well-defined computations is exhausted
by those of Turing machines.

Paul Smolensky [1988] (recipient 2005 David E. Rumelhart Prize),

On the proper treatment of connectionism, in Behavioral and Brain Sciences, 11, pp. 1-74

- Evidence is: The basic ingredients for a new computational discipline are already present



Swimming with the tide

- New computational paradigms via mathematical models seem to need more understanding of Nature than we have

- Back to a more Baconian interface with Nature . . .

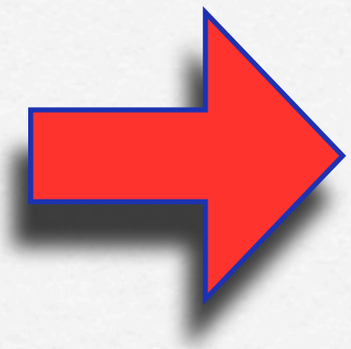
> "Some of the great physical systems to be studied as objects of control are the dynamic processes in the living organisms, especially under pathological conditions."
>
> - Boris Kogan, pioneer developer of the Soviet Union's first analog and hybrid computers, in an interview with Daniel Abramovitch, pp. 52-62 of the June 2005 issue of the IEEE Control Systems Magazine

Nature in the driving seat

- Getting intelligent machines themselves via emergence . . .

> "I used to think we'd do it by engineering. Now I believe we'll evolve them. We're likely to make thinking machines before we understand how the mind works, which is kind of backwards."
>
> – Daniel Hillis, Chief Technology Officer of Applied Minds, Inc. (and ex-Vice President, Research and Development at Walt Disney Imagineering), April 2001

- May bring operational benefits, even within standard model



Nature in the driving seat

- Idea - Ride the physical world's rich computational resources, without worrying too much about understanding the underlying rules of the game

- Approach may be limited - takes ingenuity to get a natural process to compute more than itself

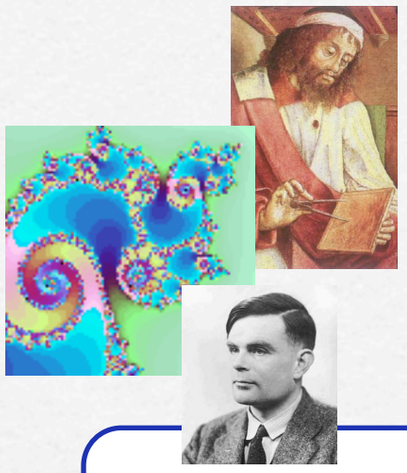
- Problem replicating the modularity and subroutines of human problem solving and the wider natural environment

Nature in the driving seat

- But - may bring practically useful results, and be the best we can do in the short to medium term

- And a potential source of Baconian understanding . . .

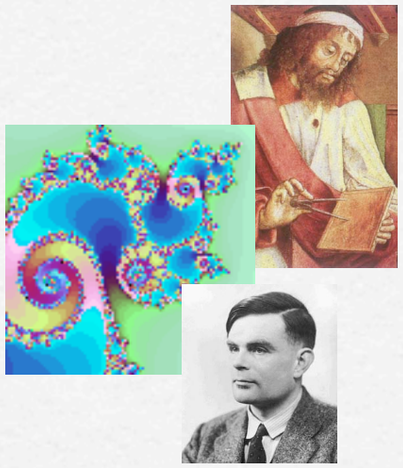
- In particular, hypercomputational suspicions are raised, the harder it is to divorce computational approaches from their real-world origins



an analogy . . .

"The domestication of horses around five or six thousand years ago brought a revolution in transportation, only achieved through a creative interaction between humans and the natural world. At that time, trying to understand the principles underlying the equine organism in order to synthesise an artificial horse was unthinkable. But a few thousand years later there was enough understanding of scientific basics to underpin the invention of the `iron horse', leading, amongst other things, to the opening up of many previously isolated parts of the world to people with no riding skills whatsoever."

– S.B.C., Definability as hypercomputational effect, Applied Mathematics and Computation, to appear

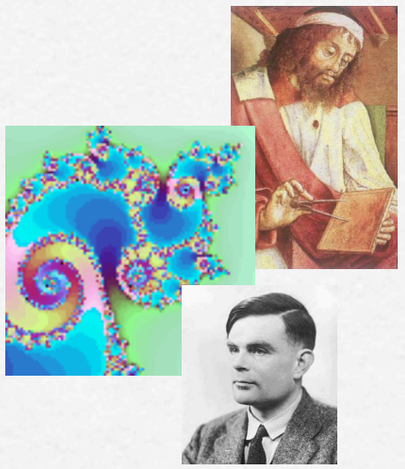


But where is the inductive structure?

- Have many exciting new computational paradigms expressing metaphors for natural processes

- - such as quantum and molecular computing, membrane computing, neural networks, cellular automata, L-systems, DNA computing, swarm and evolutionary computation, relativistic computing, and evolving real-world models like grids and the internet

- But for us algorithmic content gives rise to new emergent forms, which further feed our algorithmic appetites



Connectionist Models of Computation

- These have come a long way since Turing's [1948] discussion of 'unorganised machines', and McCulloch and Pitts [1943] early paper on neural nets

- But for Steven Pinker "... neural networks alone cannot do the job".

- And focussing on our elusive higher functionality, he points to a "kind of mental fecundity called recursion" . . .

We humans can take an entire proposition and give it a role in some larger proposition. Then we can take the larger proposition and embed it in a still-larger one. Not only did the baby eat the slug, but the father saw the baby eat the slug, and I wonder whether the father saw the baby eat the slug, the father knows that I wonder whether he saw the baby eat the slug, and I can guess that the father knows that I wonder whether he saw the baby eat the slug, and so on.

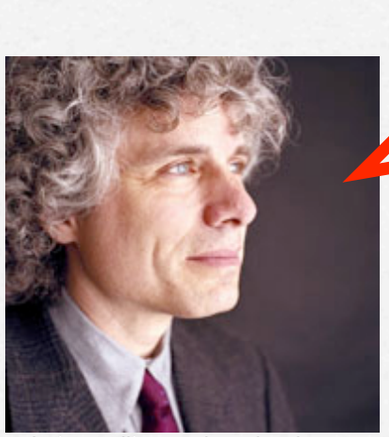


photo credit: www.harryborden.com

Steven Pinker,

How the Mind Works, W. W. Norton, New York, 1997

- Making a similar point - Damasio has a nice description of the hierarchical development of a particular instance of consciousness within the brain, interacting with some external object . . .

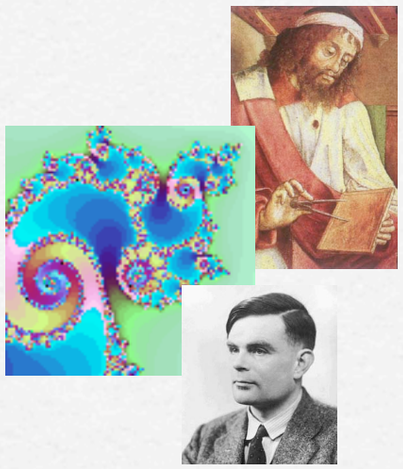
"... both organism and object are
mapped as neural patterns, in first-order maps; all of these
neural patterns can become images. ... The sensorimotor maps pertaining to the
object cause changes in the maps pertaining to the organism. ... [These] changes ... can be
re-represented in yet other maps (second-order maps) which thus represent the relationship of
object and organism. ... The neural patterns transiently formed in second-order maps
can become mental images, no less so than the neural
patterns in first-order maps."

Antonio Damasio,

The Feeling Of What Happens, Harcourt, Orlando FL, 1999

- Picture is - re-representation of neural patterns formed across some region of the brain, in such a way that they can have a computational relevance in forming new patterns

- Key conception - computational loops incorporating, in a controlled way, these 'second-order' aspects of the computation itself

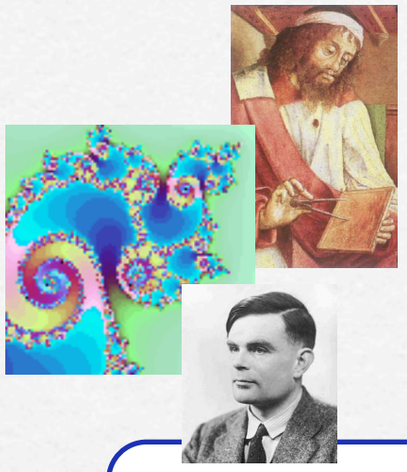


Towards a basic computational model

- Key ingredients - imaging, parallelism, interconnectivity, and a counterpart to the second-order recursions pointed to above

- Connectionist models - strong on parallelism, interconnectivity, imaging - but not recursions

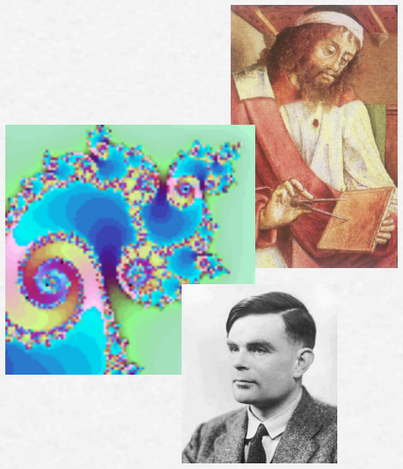
- Complex patternings of neural events emerge, but with no underlying local mechanism even though one may get a description in terms of the original structure



In the brain . . .

"As the brain forms images of an object - such as a face, a melody, a toothache, the memory of an event - and as the images of the object *affect* the state of the organism, yet another level of brain structure creates a swift nonverbal account of the events that are taking place in the varied brain regions activated as a consequence of the object-organism interaction. The mapping of the object-related consequences occurs in first-order neural maps representing the proto-self and object; the account of the *causal relationship* between object and organism can only be captured in second-order neural maps. ... one might say that the swift, second-order nonverbal account narrates a story: ***that of the organism caught in the act of representing its own changing state as it goes about representing something else.***"

- Antonio Damasio [1999], The Feeling Of What Happens, p.170

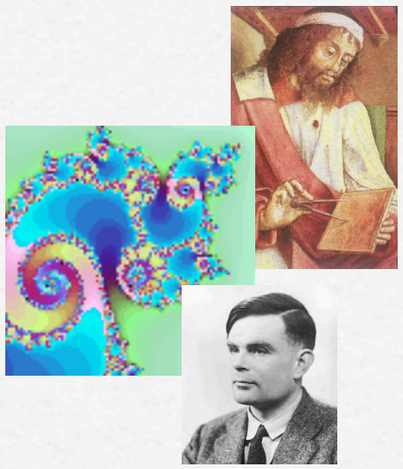


The role of external interaction

- Only makes sense to integrate external interaction into a standard model if all processes are standard

- Otherwise - algorithmic content must be signalled as the recordable link connecting non-standard information

- Now less trivial dealing with Goldin and Wegner's examples of real-world interactivity . . .

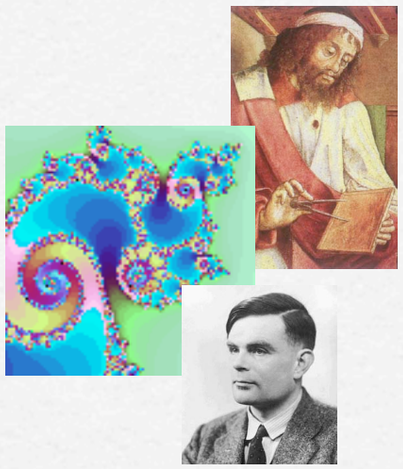


The role of external interaction

- ... taking us beyond thinking of intelligence as something that resides purely within the autonomous brain:

> "Real computational systems are not rational agents that take inputs, compute logically, and produce outputs ... It is hard to draw the line at what is intelligence and what is environmental interaction. In a sense, it does not really matter which is which, as all intelligent systems must be situated in some world or other if they are to be useful entities."

– Rodney Brooks, Intelligence Without Reason, A.I. Memo No. 1293, The A.I. Laboratory, M.I.T., Cambridge, MA, April 1991



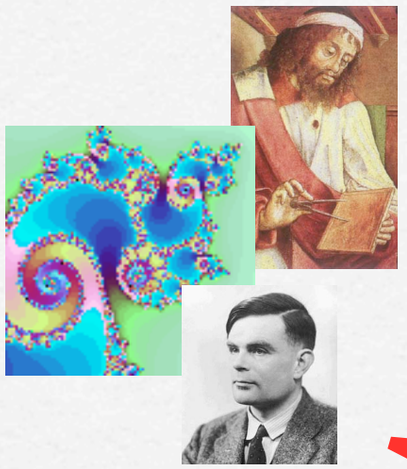
Engineering emergence

- The complexity of nature is reflected in the human brain

- The emergent brain may depend on processes which are not easily simulable over significantly shorter time-scales than those to which natural evolution is subject

- Maybe we will never build an artificial brain . . .

- . . . but may still get enough understanding of basic hypercomputational principles to build computers which do things undreamt of today



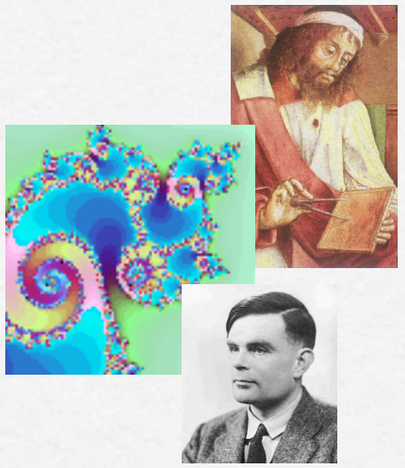
Deconstructing natural processes

Turing, 1936 -

> The real question at issue is "What are the possible processes which can be carried out in computing a [real] number?"

- not "What is a computable function?" - question freed from teleological constraints

> "Through the seventies, I became convinced that a theory of concurrency and interaction requires a new conceptual framework, not just arefinement of what we find natural for sequential computing."
>
> - Robin Milner in his 1991 Turing Award lecture

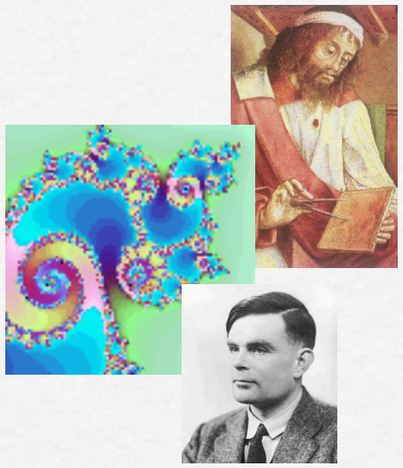


The Black Box Model Revisited

- Turing's concept of intelligence as an essentially co-operative phenomenon, and his invention of the oracle Turing machine [1939]

- And the model based on this which reflects the energy/matter dichotomy in Nature, and emergence in terms of definability (descriptions)

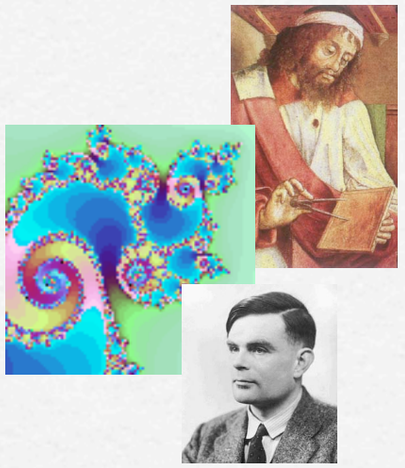
- A model which reflects scientific practice in its descriptions of the Universe in terms of extracted algorithmic content of relations over the reals



The Black Box Model Revisited

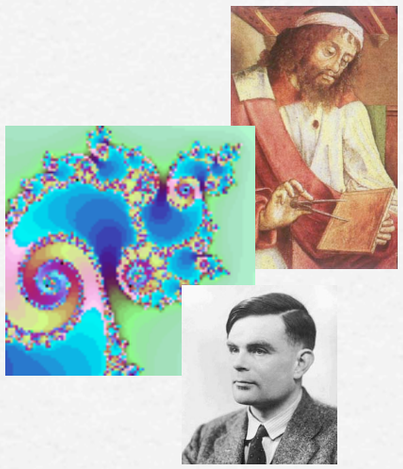
"If one abstracts from the universe its information content, structured via the basic ... fundamental laws of nature, one obtains a particular ... manifestation of the Turing universe ... , within which vague questions attain a precise analogue of quite plausible validity."

- SBC, Definability as hypercomputational effect, Applied Mathematics and Computation, to appear



Our deconstructed informational Universe

- Described *in terms of reals*
- With natural laws based on *algorithmic relations between reals*
- Emergence described in terms of *definability/invariance*
- . . . which gives rise to *new levels of algorithmic structure*
- . . . and a *fragmented scientific enterprise*

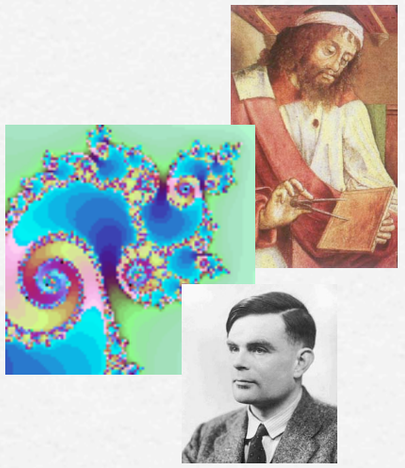


Theoretical limits on natural computation

- Turing universe – our overview of material universe – cannot uniquely describe information content

- Using our tools, it features quantum ambiguity – not true for hyperarithmetical observers!

- The human brain hosts parallel realities, which may or may not clarify

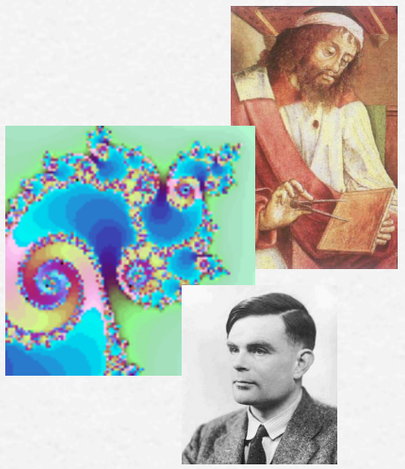


Theoretical limits on natural computation

- Identical twins analogy - in our world we may not be able to tell the difference - with better extraction of information, we could

- But - for us - there is a theoretical barrier to the quality of information we can access

- We have a universal model, but cannot build it, and can only access a small part of it



Baconian Enterprise, Cartesian Surprises . . .

- Growth of a *new interdisciplinary research culture* - particularly in Europe - bringing together computer scientists, mathematicians, physicists, natural scientists, philosophers . . .

Computability in Europe

http://www.cs.swan.ac.uk/cie06/

http://www.maths.leeds.ac.uk/cie/

The End