Constructing Interference-Minimal Networks

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Overview

1. Introduction
   - Motivation
   - What is interference?
   - Properties
2. Exact interference
   - ComputeEdgeSet
   - FulfillsProperty
3. Estimated interference
   - Definition
   - ComputeEdgeSet
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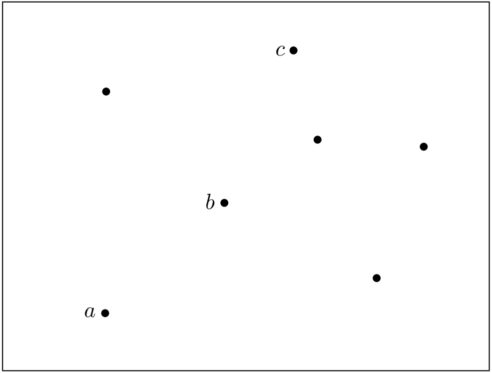

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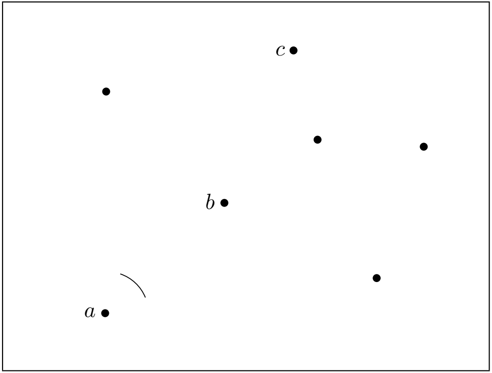

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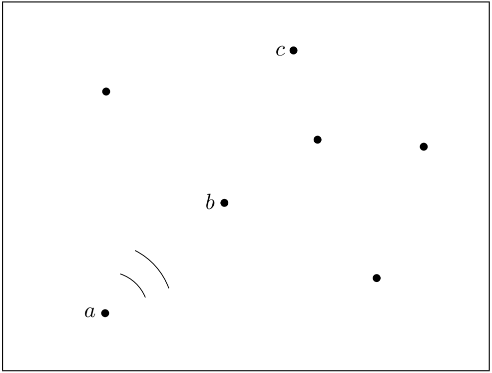

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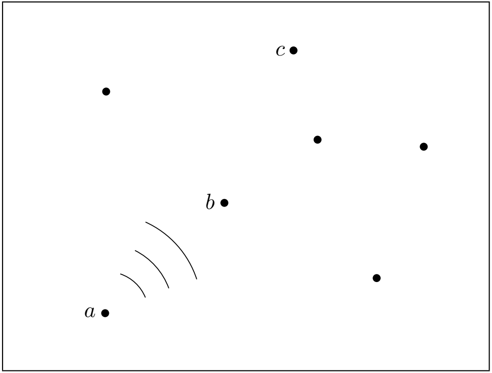

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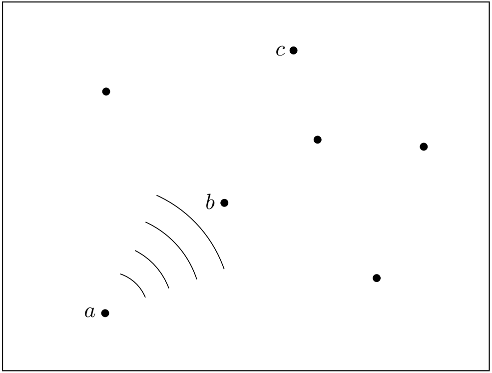

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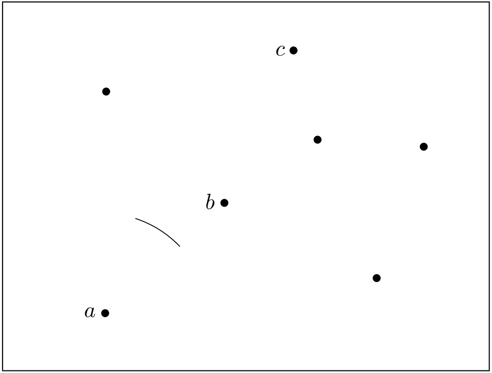

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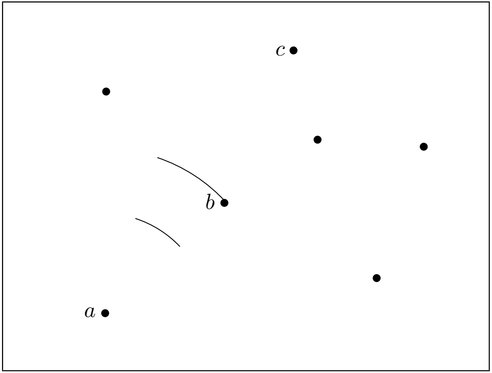

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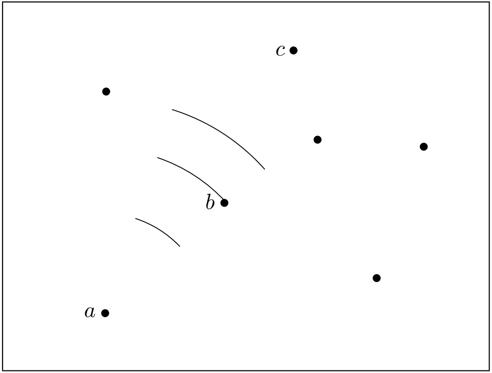

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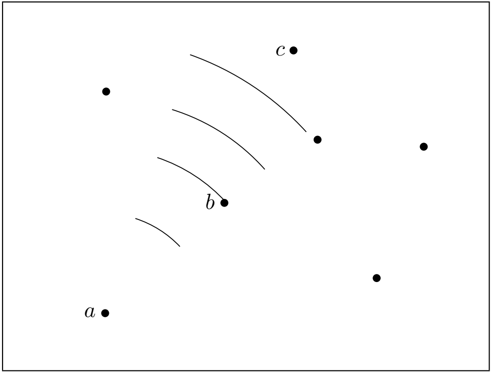

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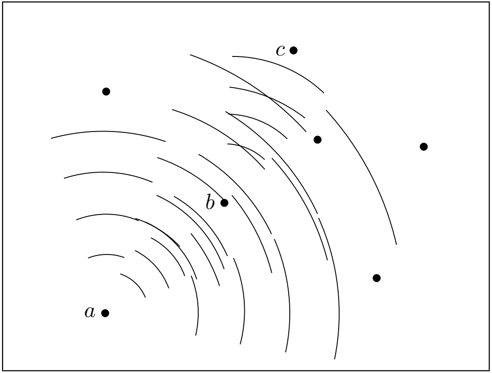

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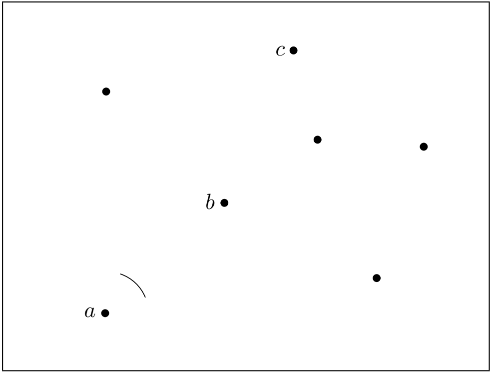

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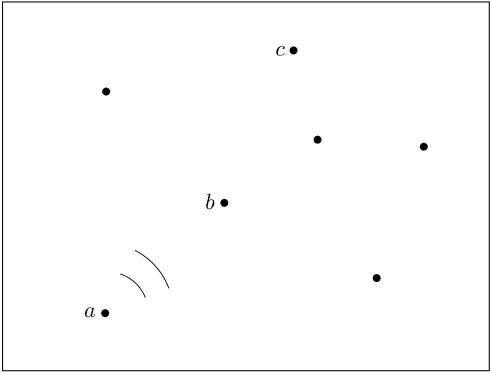

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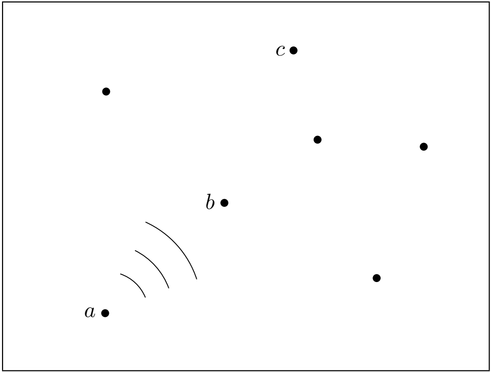

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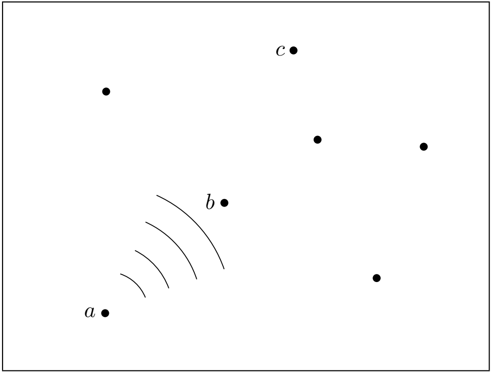

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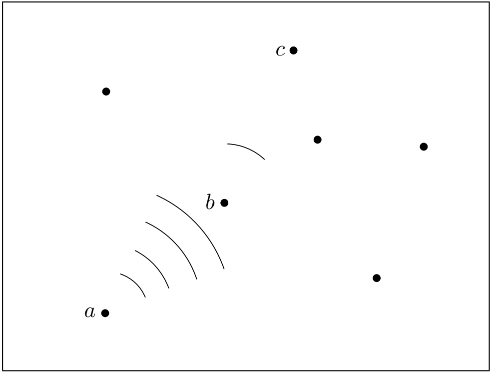

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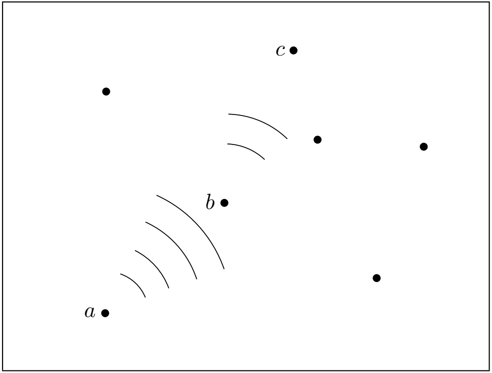

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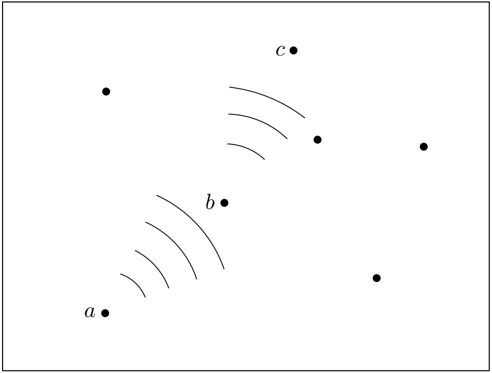

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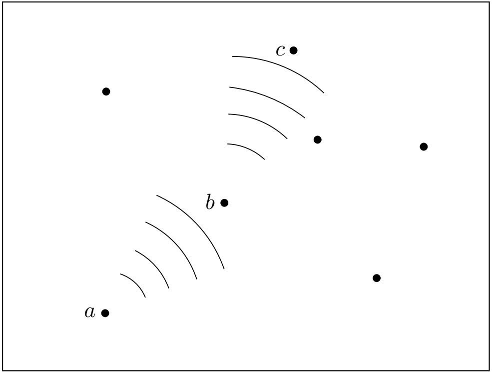
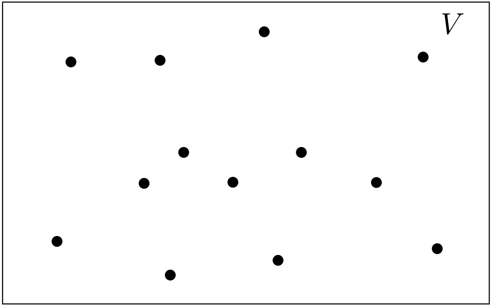


Definition of *interference*

[Burkhart et al., 2004]



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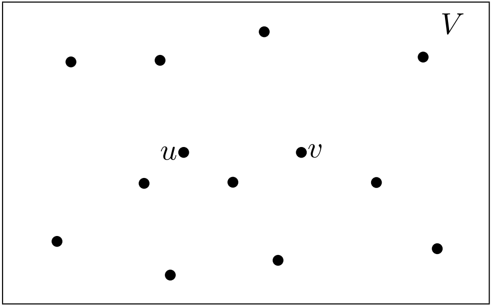

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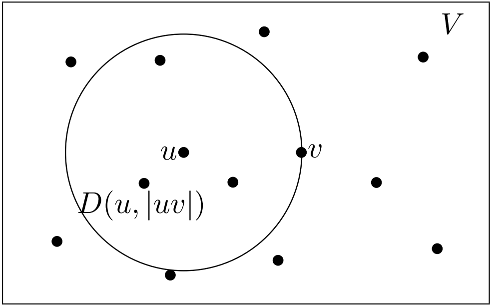

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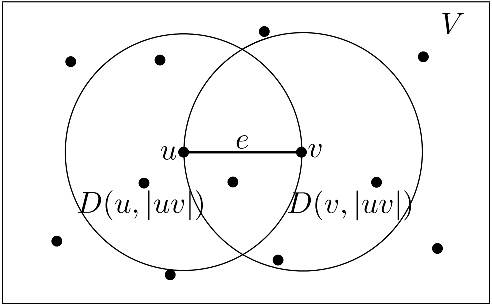

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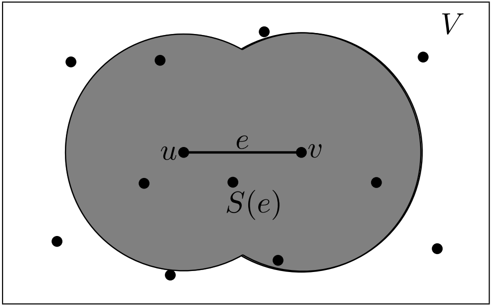

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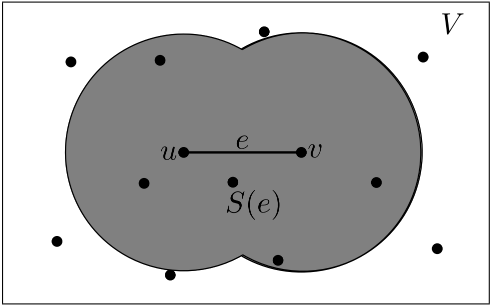

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- $\mathrm{Int}(e) := \big|V \cap S(e)\big| - 2$.

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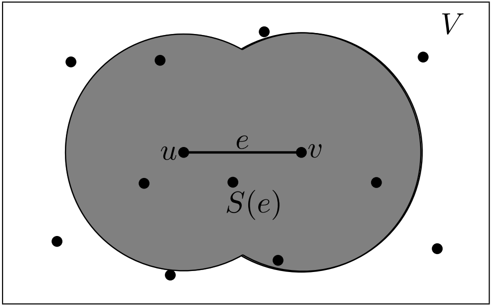

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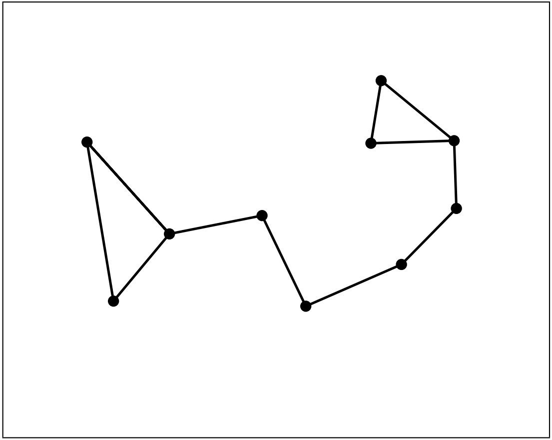
- $\mathrm{Int}(e) := \big|V \cap S(e)\big| - 2$.
- $G = (V, E)$ communication graph.
  $\mathrm{Int}(G) := \max_{e \in E} \mathrm{Int}(e)$.

Properties of graphs that we construct



connectivity

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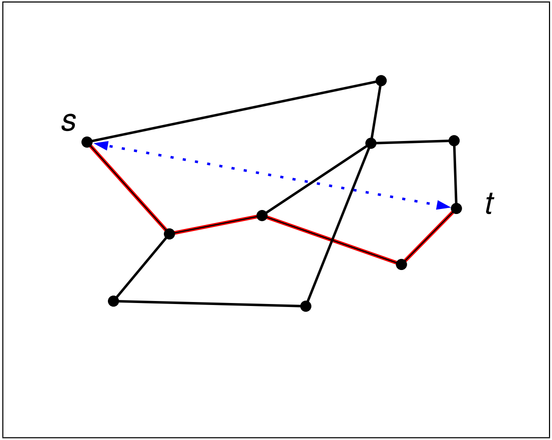

t-spanner

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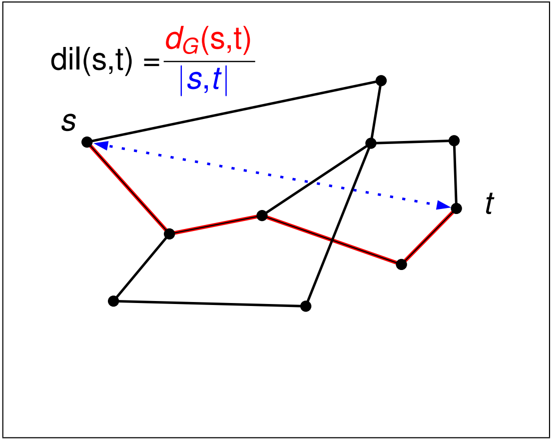

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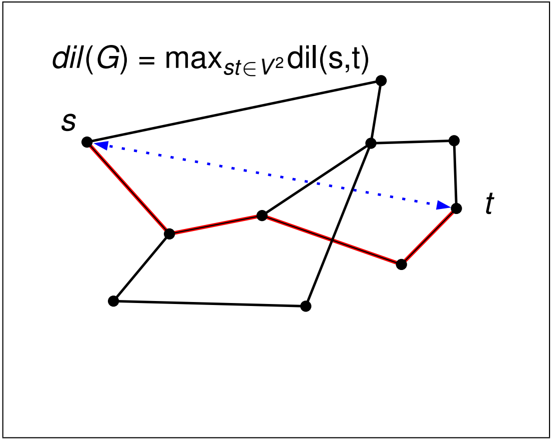

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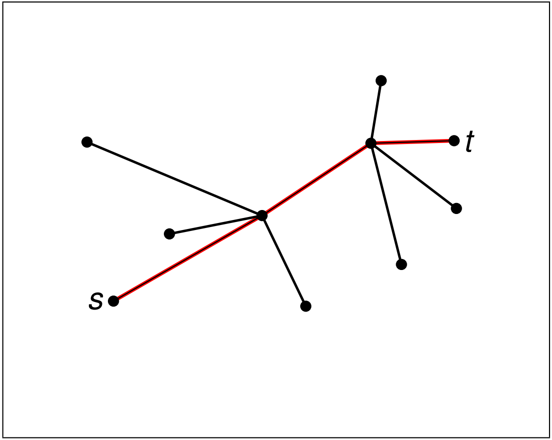

d-hop network

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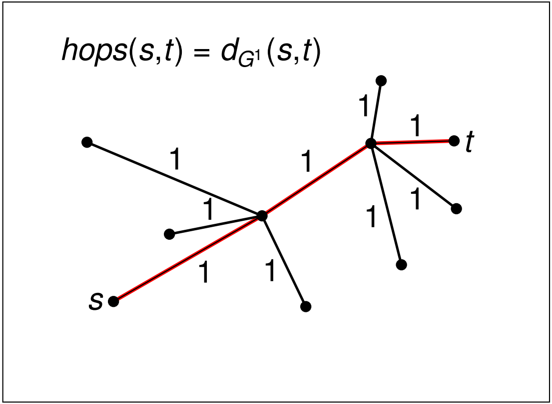
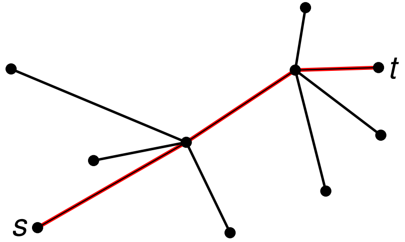


d-hop network

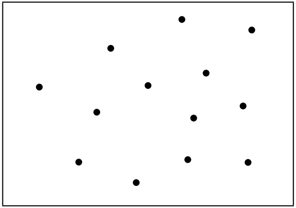
Properties of graphs that we construct

$$hops(G) = \max_{st \in V^2} hops(s,t)$$



d-hop network

Previous results



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  interference values of all edges in $O(n^{\frac{9}{4}} \text{polylog } n)$ time.

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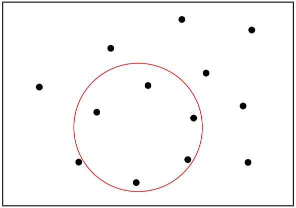

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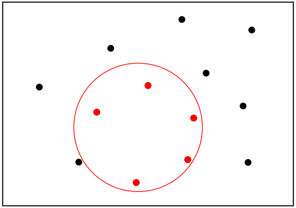

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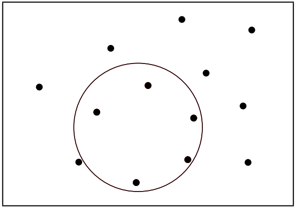

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- Interference-minimal t-spanner
  - Burkhart et al., 2004 $O(n^4)$
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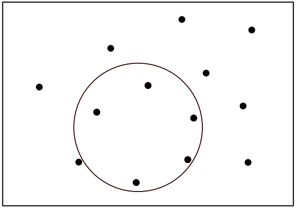

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- ! All interference values were known a priori.

Our approach

- $E_\ell := \{e \mid \mathrm{Int}(e) \leq \ell\}$, $G_\ell = (V, E_\ell)$.

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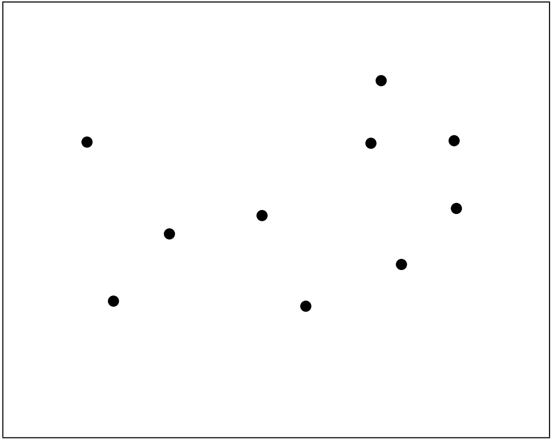
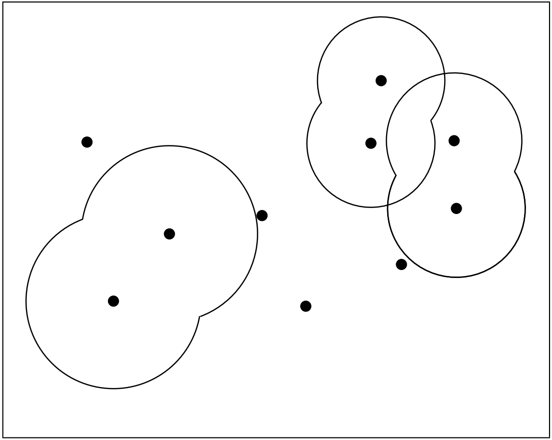
  $\Longrightarrow$ better than CRS $O(n^{\frac{9}{4}}\text{polylog } n)$ for $k \in O(n^{\frac{5}{8}})$.

Incremental approach (connectivity)



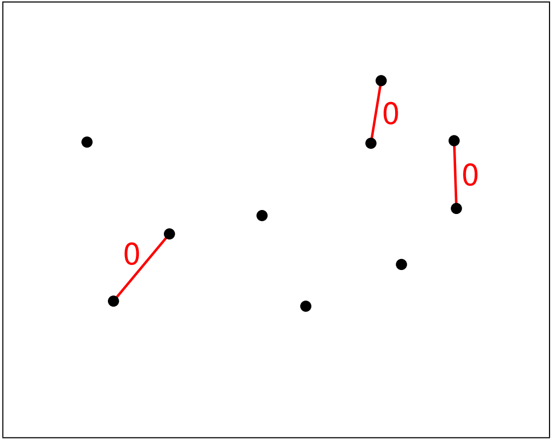
Input hosts

Incremental approach (connectivity)



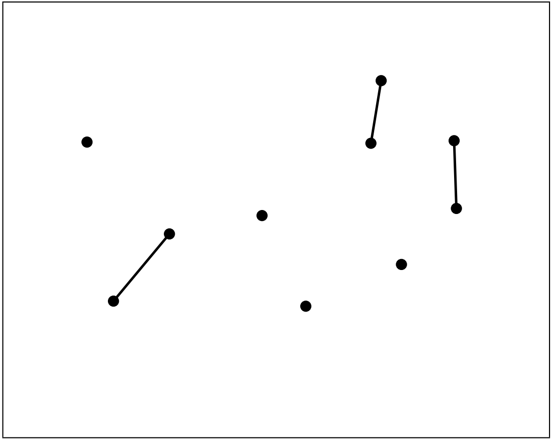
Spheres of edges e with $\mathrm{Int}(e) = 0$.

Incremental approach (connectivity)



ComputeEdgeSet E_0

Incremental approach (connectivity)



FulfillsProperty $\mathcal{P}$ (connected)?

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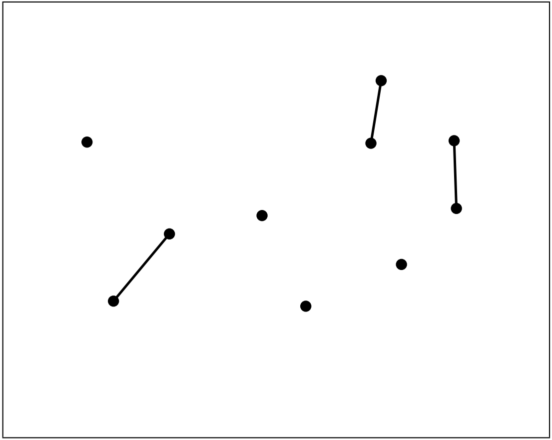

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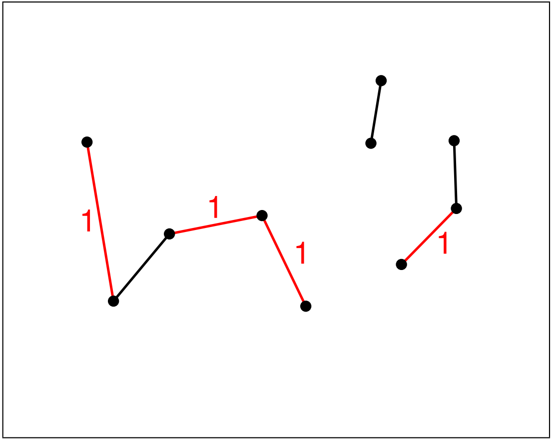
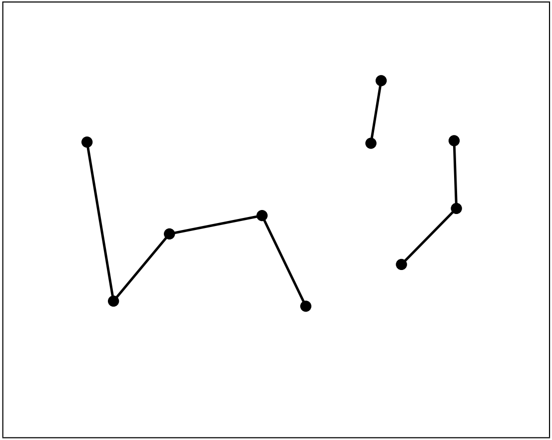


ComputeEdgeSet E_1

Incremental approach (connectivity)



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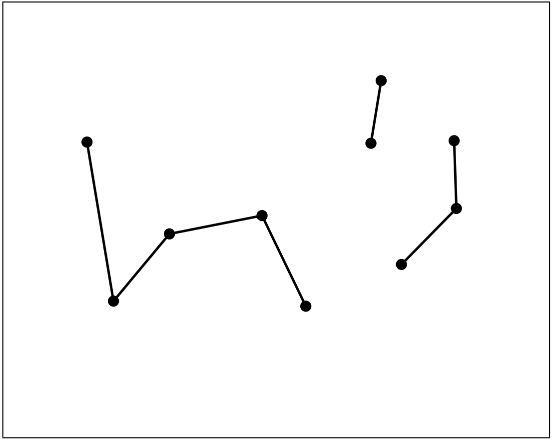

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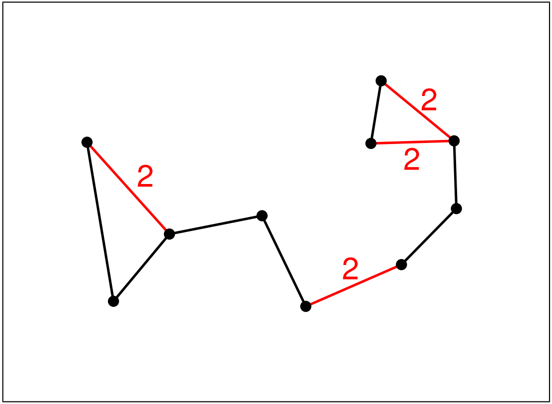
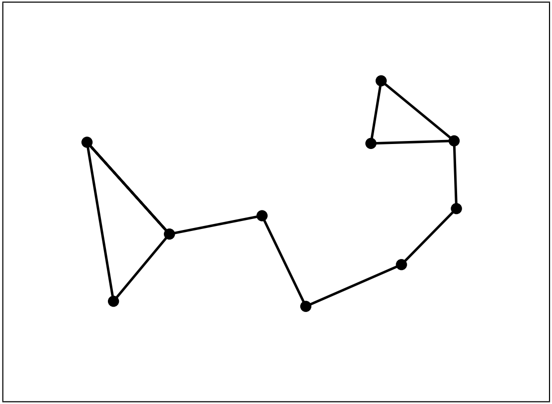


ComputeEdgeSet E_2

Incremental approach (connectivity)



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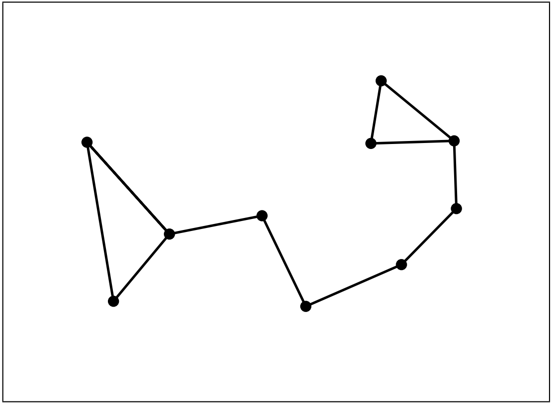

Yes.

Overall proceeding

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Naive **Algorithm**

1. For $\ell = 0$ to n do
2. ComputeEdgeSet E_ℓ
3. FulfillsProperty G_ℓ

k=27, n=128: 0,1,2,3,...,27 $\rightarrow O(k)$ steps.

Overall proceeding

Given: Hosts V and a graph property $\mathcal{P}$, $n = |V|$.
Find: k, the smallest integer such that G_k has property $\mathcal{P}$.

Better **Algorithm**

1. Apply binary search to find k
2. ComputeEdgeSet E_ℓ
3. FulfillsProperty G_ℓ

k=27, n=128: 0,128,64,32,16,24,28,26,27 $\rightarrow O(\log n)$ steps.

Overall proceeding

Given: Hosts V and a graph property $\mathcal{P}$, $n = |V|$.
Find: k, the smallest integer such that G_k has property $\mathcal{P}$.

Best **Algorithm**

1. Apply exponential and binary search to find k
2. ComputeEdgeSet E_ℓ
3. FulfillsProperty G_ℓ

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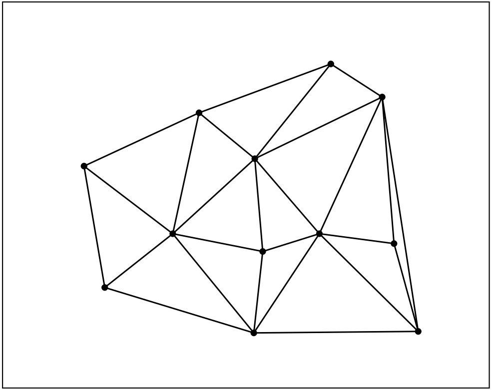
k=27, n=128: 0,1,2,4,8,16,32,24,28,26,27 $\rightarrow O(\log k)$ steps.

It remains to implement ComputeEdgeSet E_ℓ and FulfillsProperty G_ℓ !

Overview

ComputeEdgeSet E_ℓ : Excursion Delaunay Edge.



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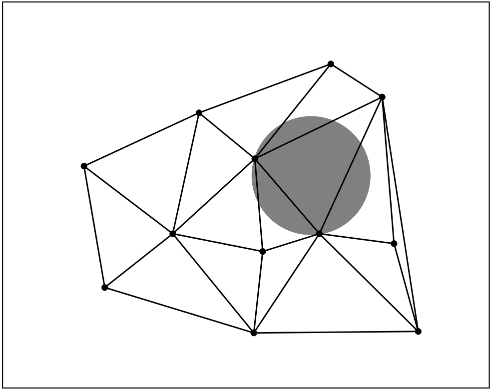

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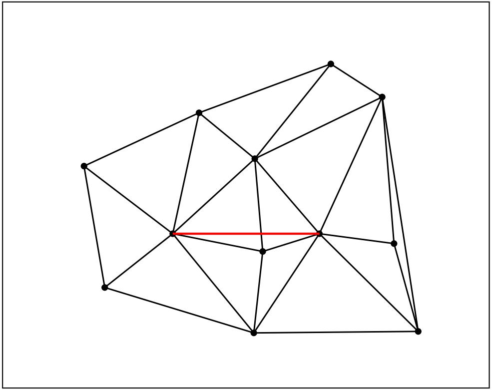

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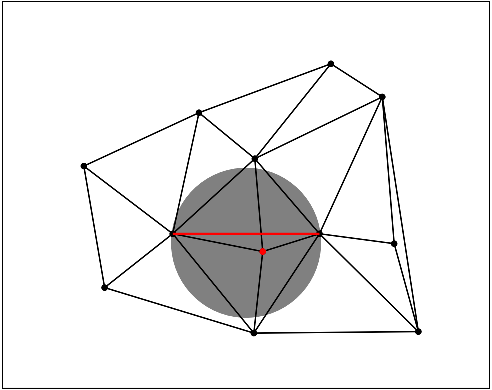

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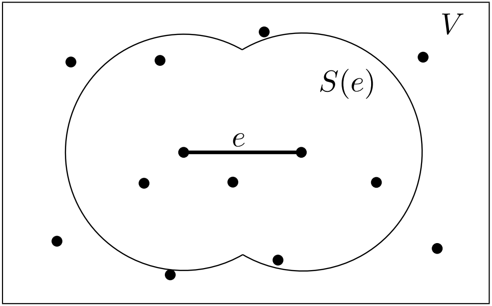

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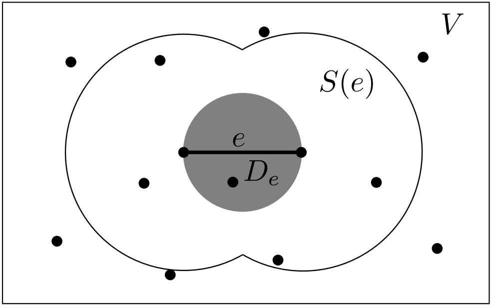

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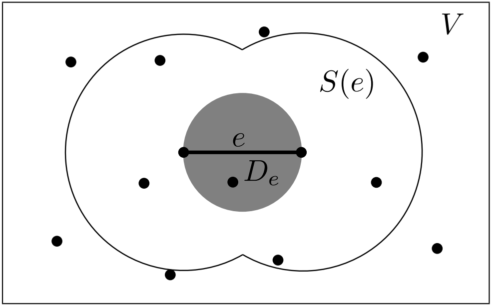

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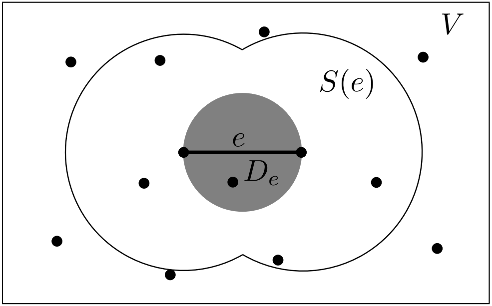

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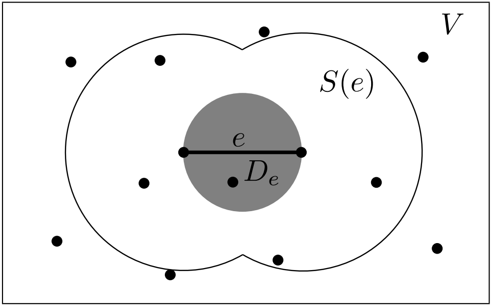

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- DG_ℓ can be computed in $O(n\ell \log \ell + n \log n)$ exp. time. [GHK02]
- $|DG_\ell| = O(n\ell)$. $\implies$ can compute E_ℓ in $O(n\ell^2 + n \log n)$.

FulfillsProperty G_ℓ

Is G_ℓ connected?

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- $O(n\ell)$ time

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Is G_ℓ a t-spanner?

- all pairs shortest path computation, weight$(u, v) = |uv|$
- check for all pairs $d_{G_\ell}(u, v) \leq t|uv|$
- n times Dijkstra's shortest path algorithm $\rightarrow\ O(n^2(k + \log n))$

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Is G_ℓ a d-hop network?

FulfillsProperty G_ℓ

Is G_ℓ connected?

- breadth first search
- check if all hosts were reached
- $O(n\ell)$ time

Is G_ℓ a t-spanner?

- all pairs shortest path computation, weight$(u, v) = |uv|$
- check for all pairs $d_{G_\ell}(u, v) \leq t|uv|$
- n times Dijkstra's shortest path algorithm $\rightarrow\ O(n^2(k + \log n))$

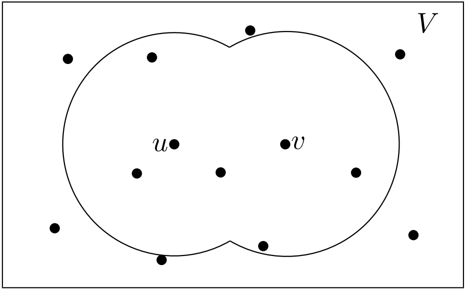
Is G_ℓ a d-hop network?

- all pairs shortest path computation, weight$(u, v) = 1$
- check for all pairs $d_{G_\ell}(u, v) \leq d$
- $O(n^2 \log n)$ exp. time [MT87]

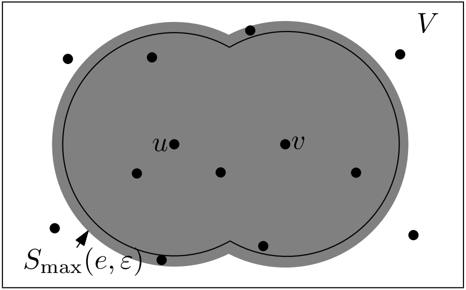
Overview

1. Introduction
   - Motivation
   - What is interference?
   - Properties
2. Exact interference
   - ComputeEdgeSet
   - FulfillsProperty
3. Estimated interference
   - Definition
   - ComputeEdgeSet
   - FulfillsProperty

Definition of *estimated* interference



Definition of *estimated* interference



- $S_{\max}(e, \varepsilon) =$ "$(1 + \varepsilon) \cdot S(e)$"

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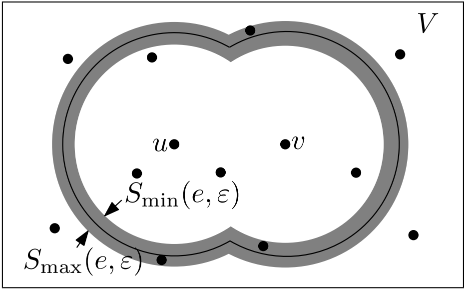

- $S_{\max}(e,\varepsilon) =$ "$(1+\varepsilon)\cdot S(e)$"
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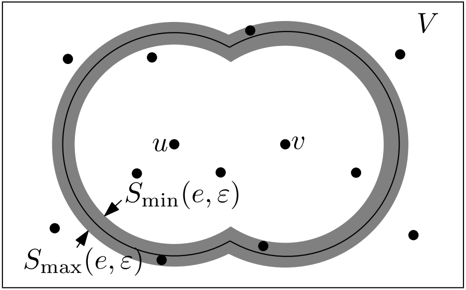
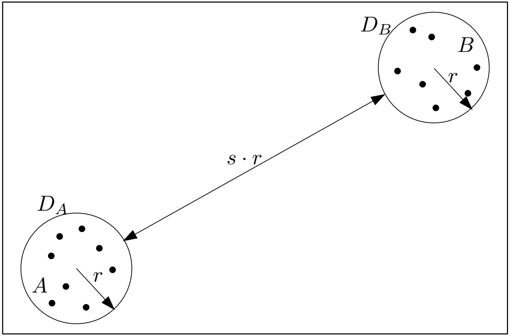
Definition of *estimated* interference



- $S_{\max}(e, \varepsilon) =$ "$(1 + \varepsilon) \cdot S(e)$"
- $S_{\min}(e, \varepsilon) =$ "$(1 - \varepsilon) \cdot S(e)$"
- $\tilde{\mathrm{Int}}$ is an ε-valid *interference estimation* of e if

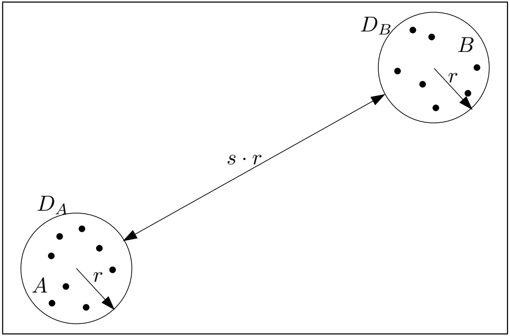
$$|S_{\max}(e, \varepsilon) \cap V| - 2 \;\geq\; \tilde{\mathrm{Int}} \geq |S_{\min}(e, \varepsilon) \cap V| - 2.$$

Well Separated Pair Decomposition [CK95]



A *well-separated* pair $\{A, B\}$ of point sets.

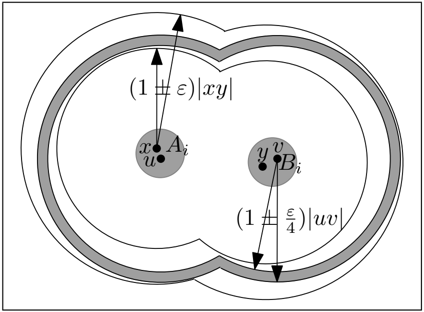
Well Separated Pair Decomposition [CK95]



A *well-separated* pair $\{A, B\}$ of point sets.

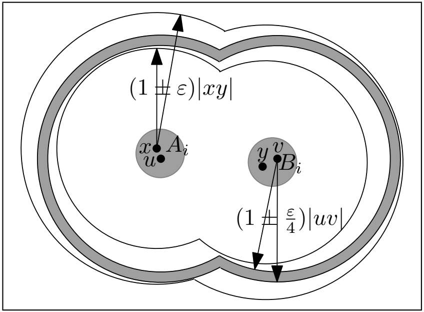
There is a WSPD $\{A_1, B_1\}, \ldots, \{A_k, B_k\}$ with $k = O(s^2 n)$ well-sep. pairs such that every point pair is separated by exactly one $\{A_i, B_i\}$.

ComputeEdgeSet E_ℓ (estimated!)



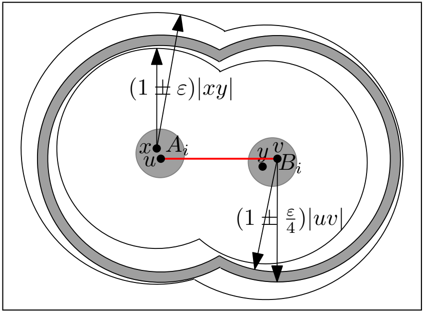
Goal: Faster computation than in the exact case. $O(n\ell^2)$

ComputeEdgeSet E_ℓ (estimated!)



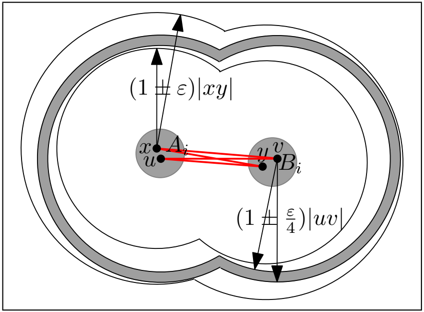
Approach: Preprocess ε-valid interference estimations for all edges.

ComputeEdgeSet E_ℓ (estimated!)



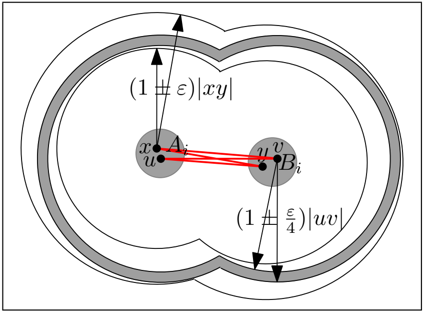
Idea: For each $\{A_i, B_i\}$ compute only one $\frac{\varepsilon}{4}$-valid interference estimations for an arbitrary edge $\{u, v\}$ ...

ComputeEdgeSet E_ℓ (estimated!)



Idea: For each $\{A_i, B_i\}$ compute only one $\frac{\varepsilon}{4}$-valid interference estimations for an arbitrary edge $\{u, v\}$ ...and assign its value to all edges that connect $\{A_i, B_i\}$.

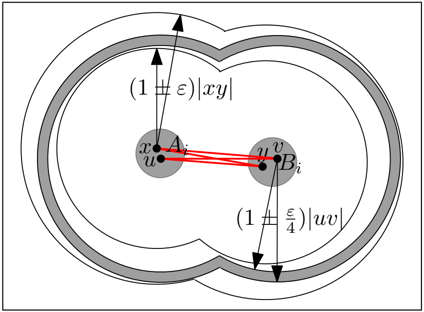
ComputeEdgeSet E_ℓ (estimated!)



Running time: $O(\frac{n}{\varepsilon^2} \cdot (\frac{1}{\varepsilon} + \log n))$ preprocessing.

approximate range counting [AM00]

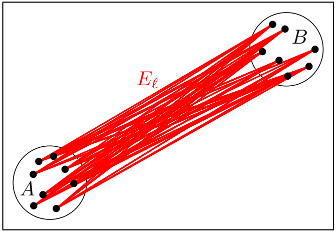
ComputeEdgeSet E_ℓ (estimated!)



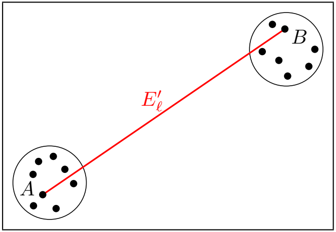
Running time: $O(\frac{n}{\varepsilon^2} \cdot (\frac{1}{\varepsilon} + \log n))$ preprocessing.

$\Longrightarrow O(\frac{n}{\varepsilon^2})$ to compute one E_ℓ exact: $O(n\ell^2)$.

FulfillsProperty G_ℓ (estimated)

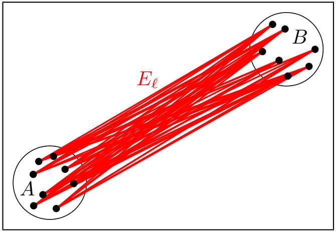


$$|E_\ell| = O(n^2)$$



$$|E'_\ell| = O(\frac{n}{\varepsilon^2})$$

FulfillsProperty G_ℓ (estimated)



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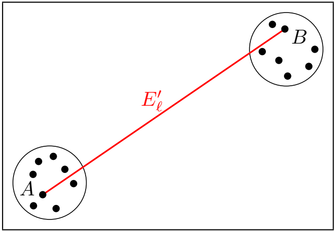
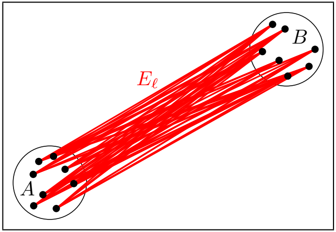


$|E'_\ell| = O(\frac{n}{\varepsilon^2})$

- Property testing (connectivity) in $G'_\ell = (V, E'_\ell)$.

FulfillsProperty G_ℓ (estimated)



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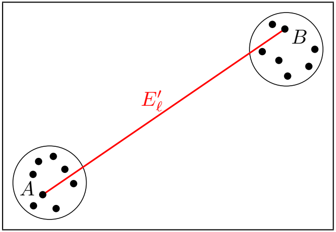
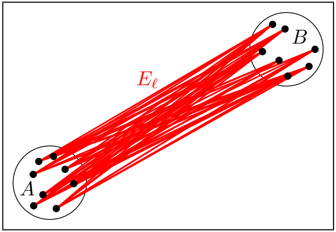
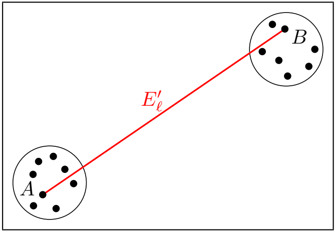


$|E'_\ell| = O(\frac{n}{\varepsilon^2})$

- Property testing (connectivity) in $G'_\ell = (V, E'_\ell)$.
- **Theorem:** G_ℓ connected $\Longleftrightarrow$ G'_ℓ connected.

FulfillsProperty G_ℓ (estimated)





$|E_\ell| = O(n^2)$ $\qquad$ $|E'_\ell| = O(\frac{n}{\varepsilon^2})$

- Property testing (connectivity) in $G'_\ell = (V, E'_\ell)$.
- **Theorem:** G_ℓ connected $\Longleftrightarrow$ G'_ℓ connected.
- $\Longrightarrow$ BFS in G'_ℓ runs in $O(\frac{n}{\varepsilon^2})$ time.

Tool boxes

Exact case

	connectivity	t-spanner	d-hop
CompEdgeSet E_ℓ	order-ℓ Delaunay triangulation		
FulfillsProperty G_ℓ	BFS	all pairs shortest path	

Estimated case

	connectivity	t-spanner	d-hop
CompEdgeSet E'_ℓ	Well-separated pair decomposition		
FulfillsProperty G_ℓ	BFS	approx.dilation	implicitBFS

Running times

Exact case

	connectivity	t-spanner	d-hop
CompEdgeSet E_ℓ	$O(n\ell^2 + n\log n)$		
FulfillsProperty G_ℓ	$O(n\ell)$	$O(n^2 \log n)$	

Estimated case

	connectivity	t-spanner	d-hop
CompEdgeSet E'_ℓ	preproc. $O(\frac{n}{\varepsilon^2}[\frac{1}{\varepsilon} + \log n])$, query $O(\frac{n}{\varepsilon^2})$		
FulfillsProperty G_ℓ	$O(\frac{n}{\varepsilon^2})$	$O(\frac{n^2}{\varepsilon^4} \log n)$	$O(\frac{n^2}{\varepsilon^2})$

Challenges

- Find better algorithms for $k \in \Omega(n^{\frac{5}{8}})$, i.e. faster than $O(n^{\frac{9}{4}} \text{polylog } n)$. (CRS)
- Find other desirable properties for communication graphs.