A Polynomial-Time Checkable Sufficient Condition for Deadlock-Freedom of Component-Based Systems

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joint work with Mila Majster-Cederbaum & Moritz Martens

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The Setting

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- Deadlock-Detection in Component-Based Systems is NP-hard [C. Minnameier. Submitted for publication in *IPL*.]
- We give a polynomial-time computable sufficient condition for deadlock-freedom.

Part 1:
Interaction Systems & Deadlocks

An Interaction System is a Tuple $Sys = (K, \{A_i\}_{i \in K}, C, \{T_i\}_{i \in K})$

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  Every action of every component has to occur in at least one connector.
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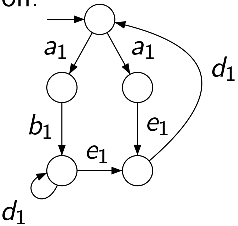
  Connectors are maximal w.r.t. set inclusion.
- The local (labeled) transition systems $\{T_i\}_{i \in K}$
  Every node has at least one outgoing edge.



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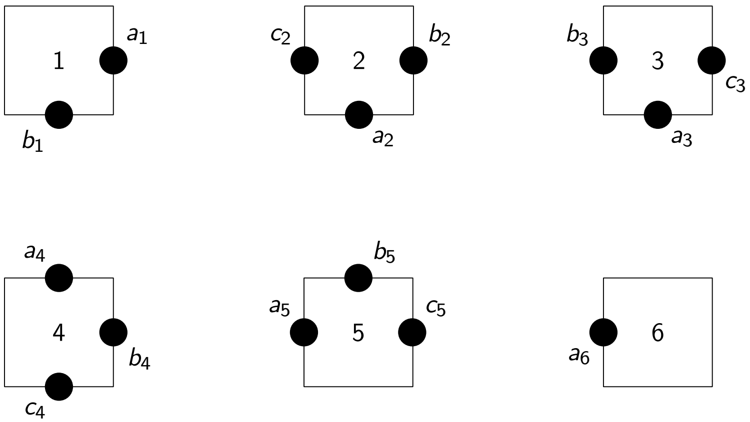

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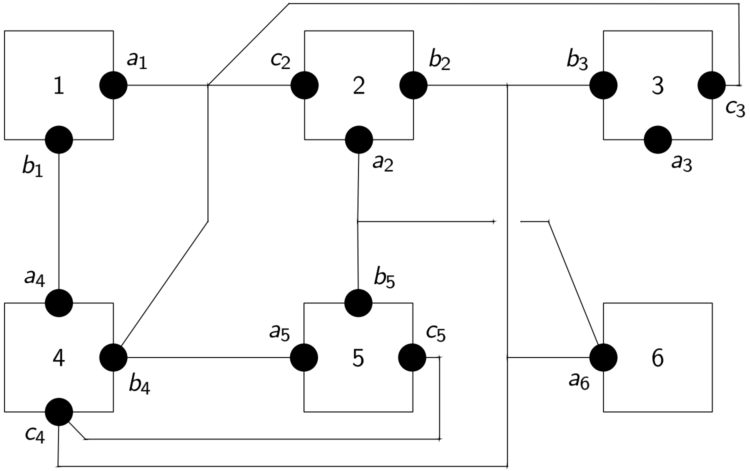

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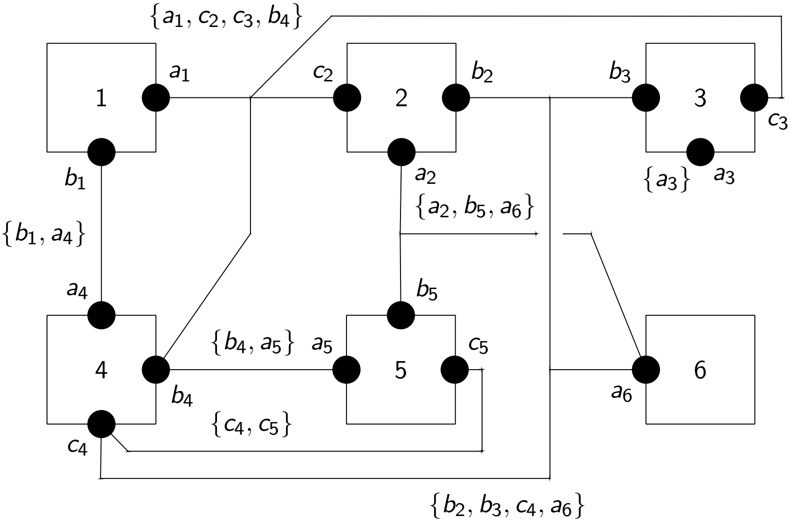

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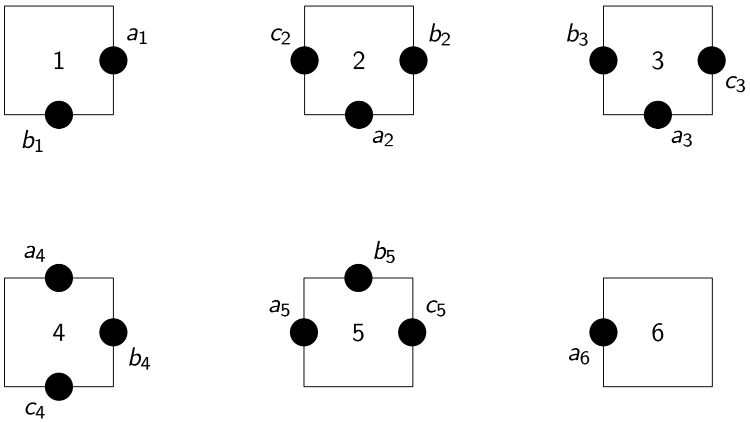

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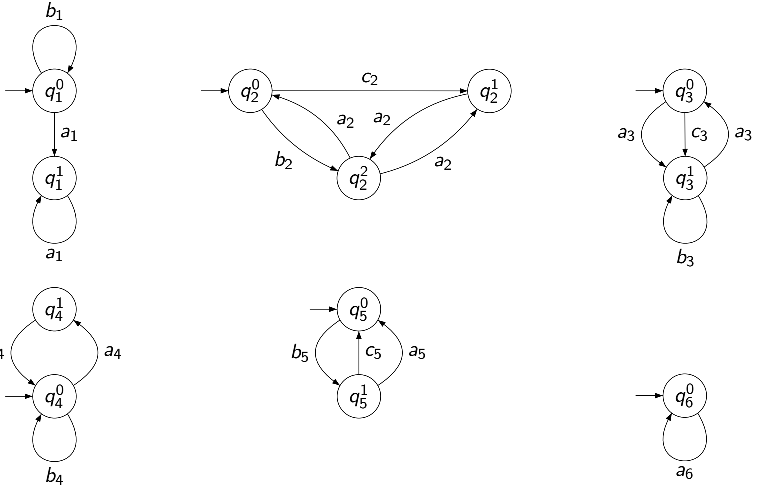

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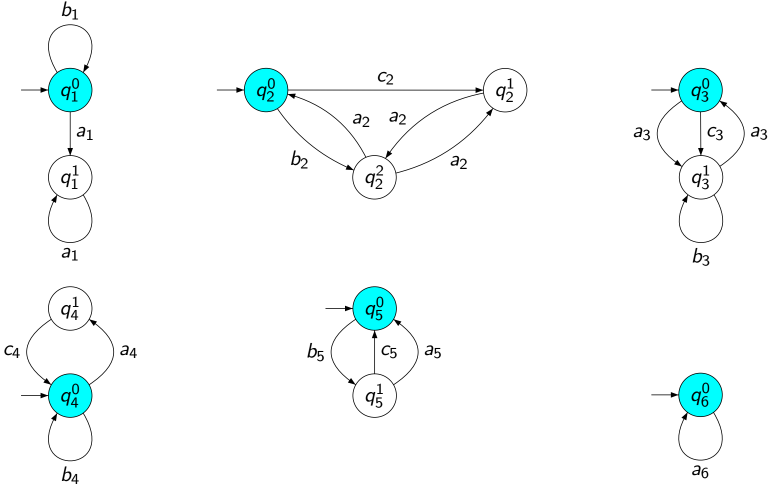

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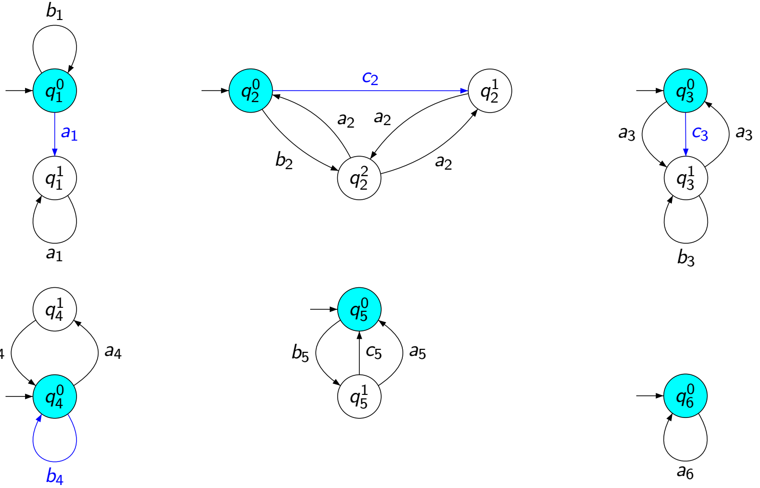

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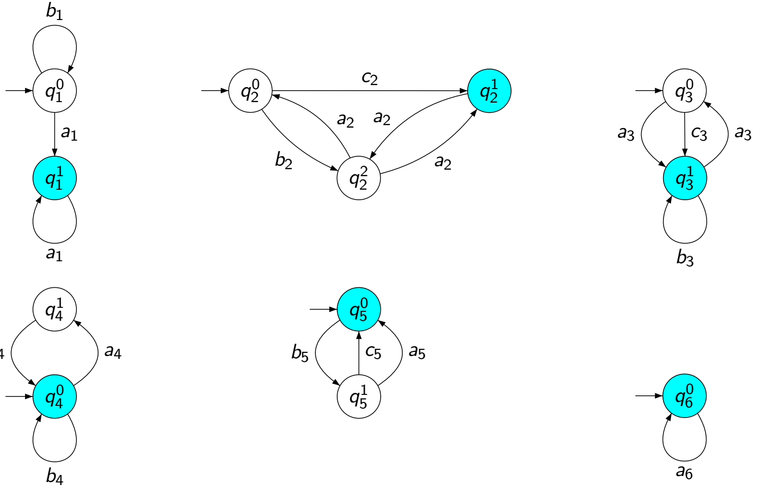

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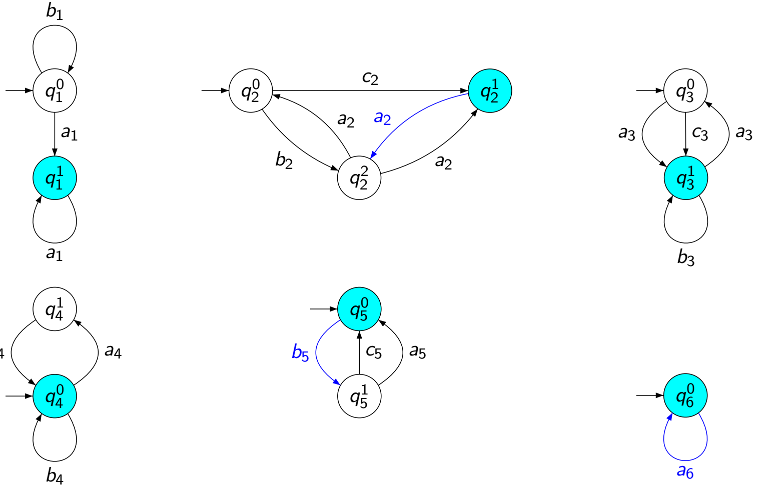

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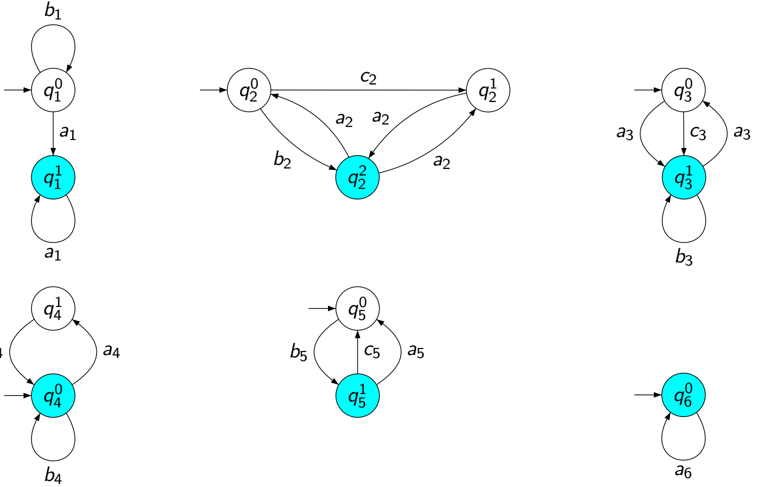

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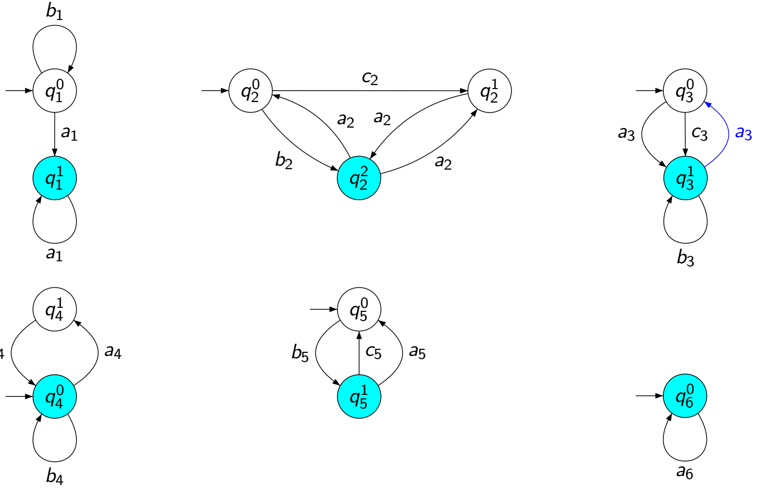

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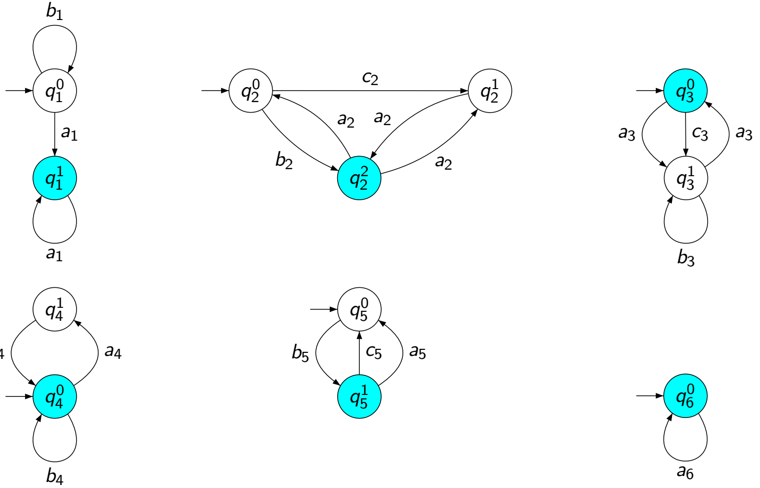

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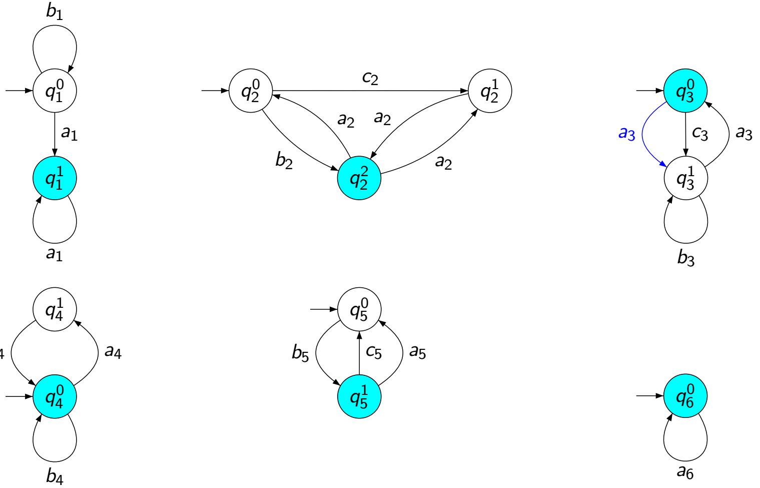

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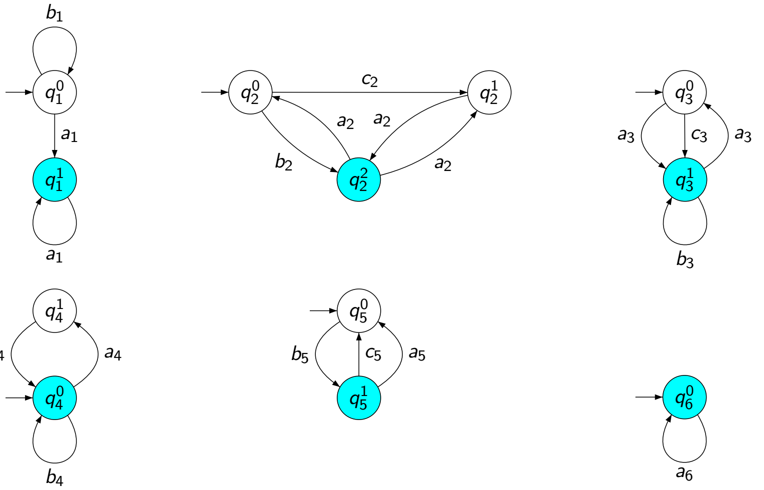

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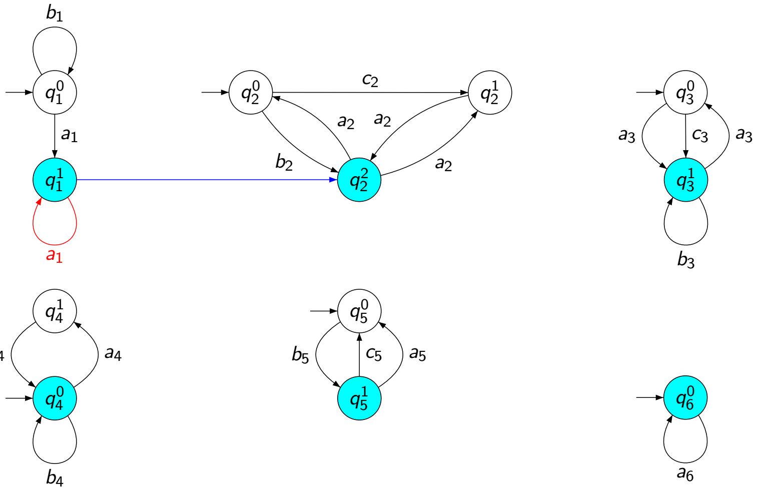

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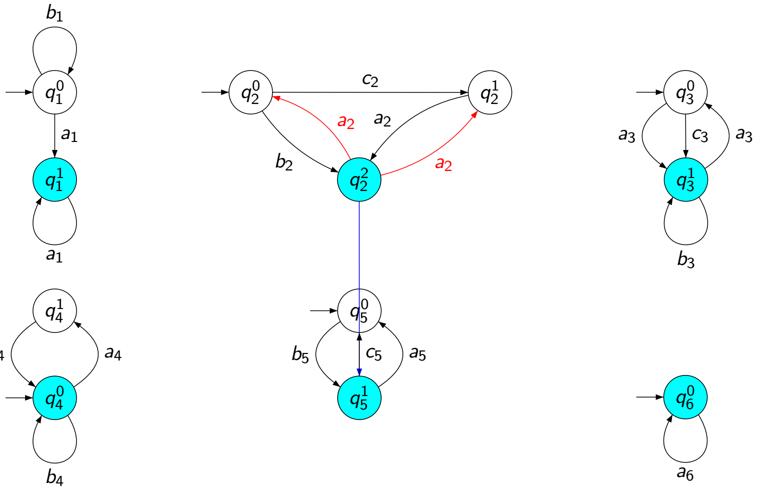

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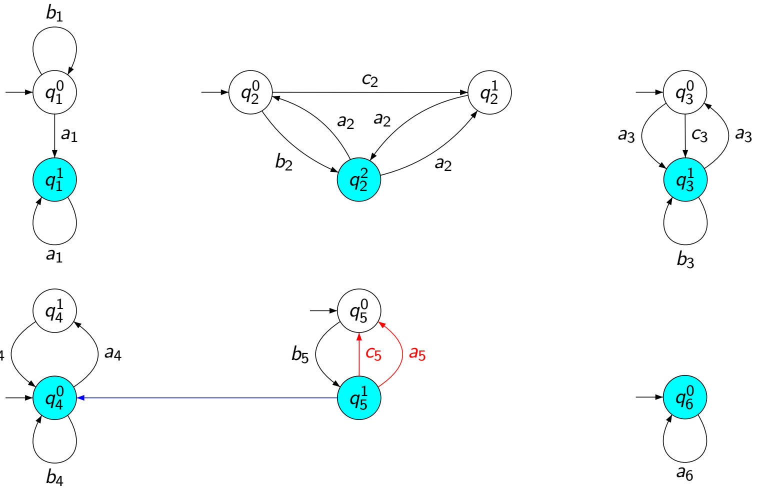

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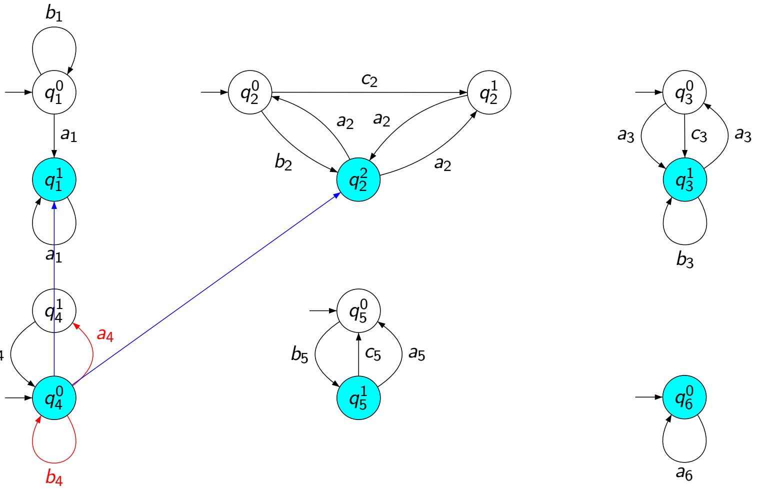

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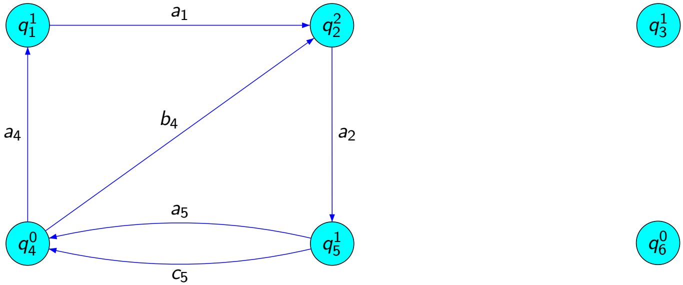
A local Deadlock - Successor-Closed Subgraph



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Let $q = (q_1, \ldots, q_n) \in Q$ be a global state.

We say that some non-empty set $D = \{j_1, j_2, \ldots, j_k\} \subseteq K$ of components is in *local deadlock* in q iff

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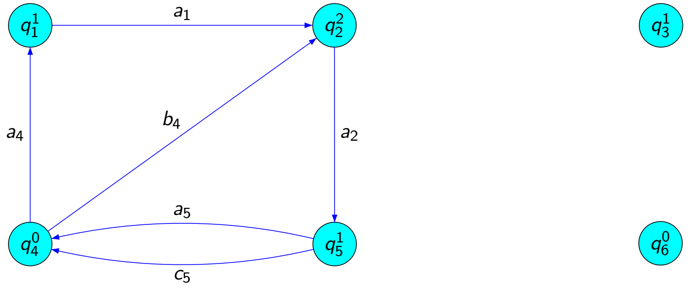
Deadlock-Detection is NP-hard!

Part 2:
Proving Deadlock-Freedom in Polynomial Time

A Successor-Closed Subgraph implies a Cycle



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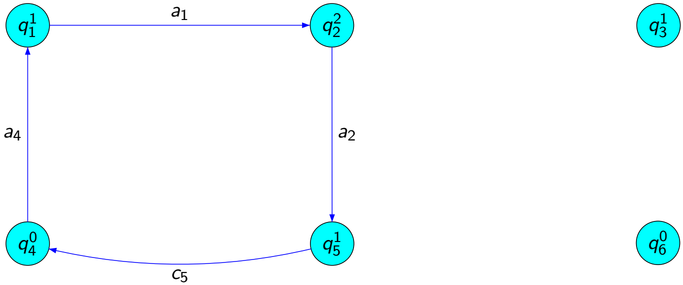
No Cylce (in any reachable global state)

$\Rightarrow$ **No Deadlock (in any reachable global state)**



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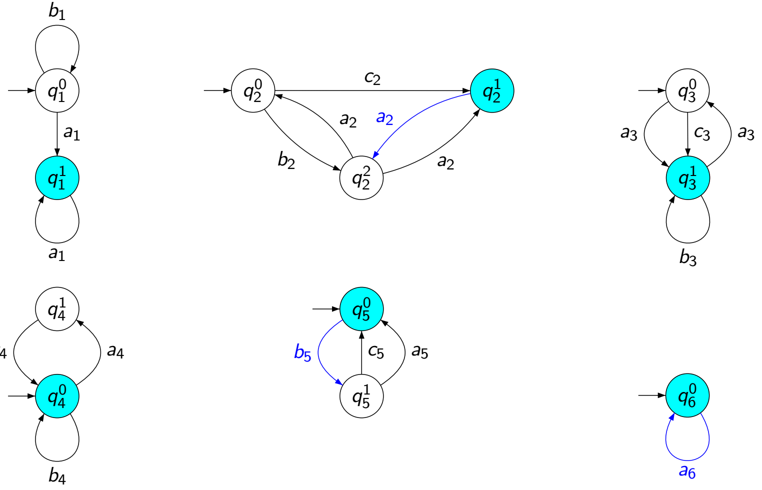
- Then there has to be a component (namely one that participates in the cycle) that just (properly) changed its local state
- So there has to be a component that just (properly) changed its local state in such a way that:
  - it waits for some component and
  - it is waited for by some component

The last global Transition before the Cycle occured



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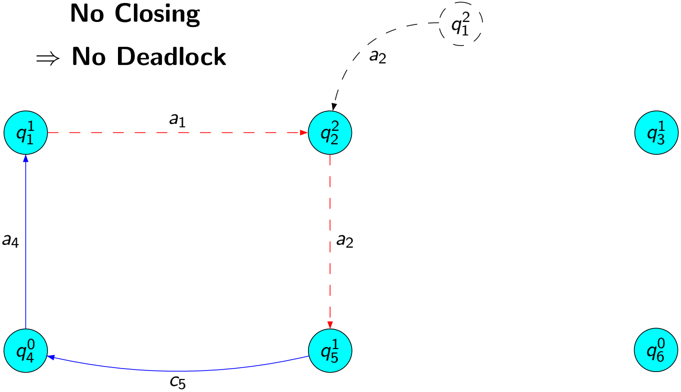
A Cycle implies a Closing

No Closing

$\Rightarrow$ **No Deadlock**



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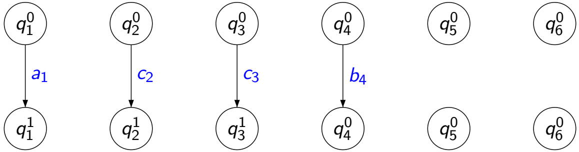

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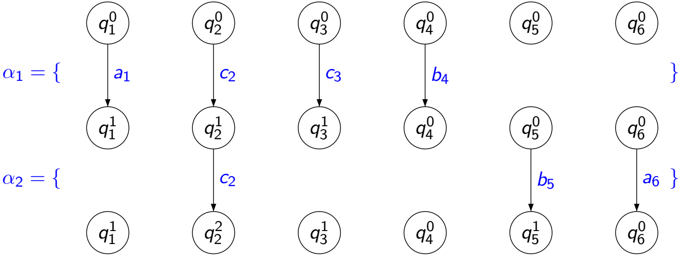

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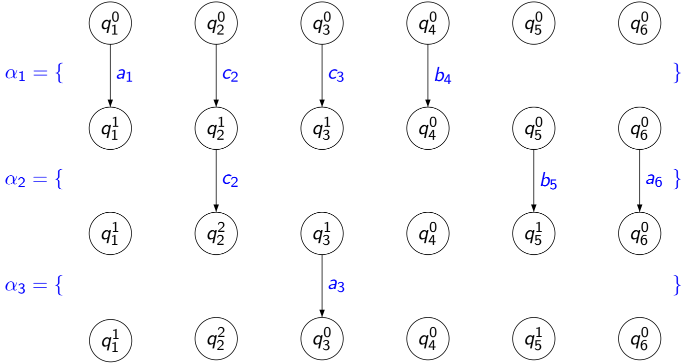

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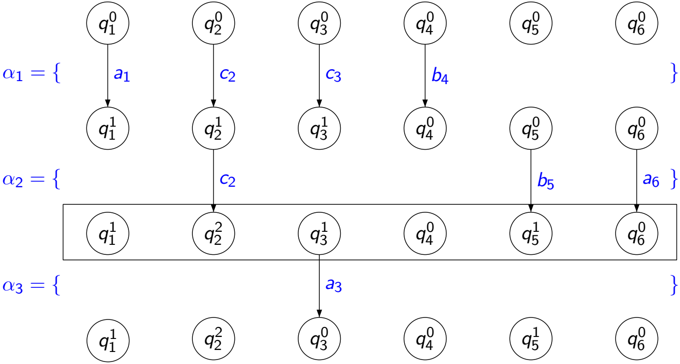

The state where the Cycle occured for the first time.

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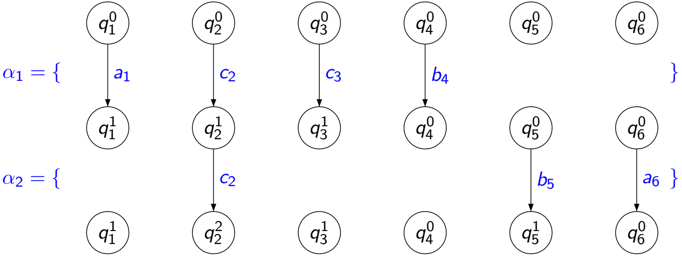

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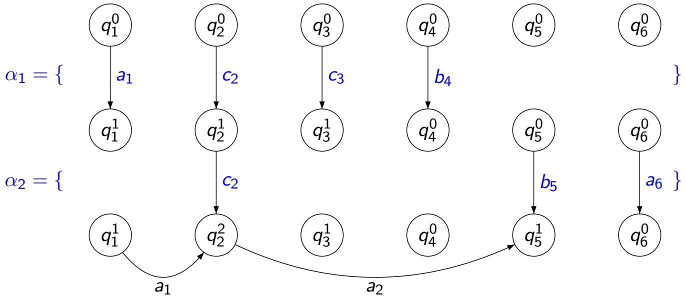
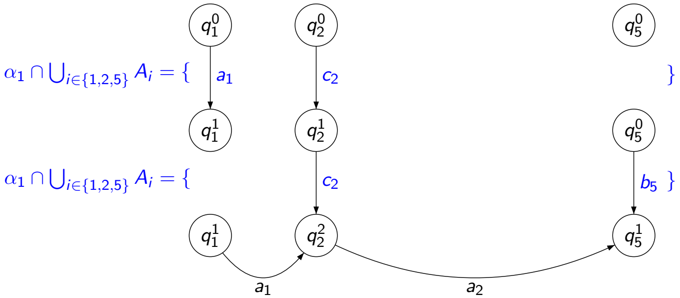


Detecting this Closing in a Subsystem



The witness of the potential formation of a cycle is still present after restricting the connectors to the action sets of the observed components.

Complexity and Parametrization

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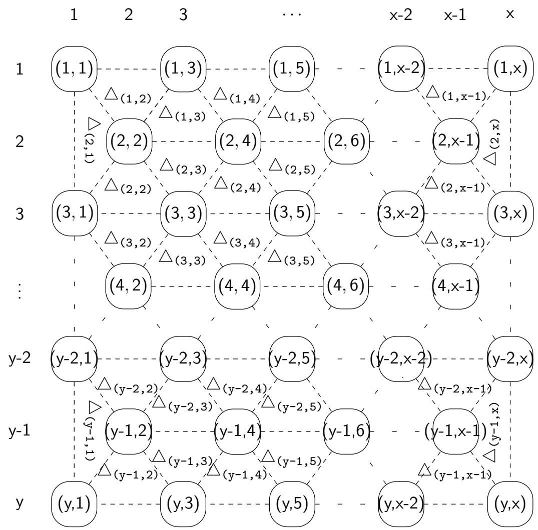
- Each such subsystem has at most m^d states, where m is the size of a largest local transition system.
- To check whether there is a component that performed a proper state change and is now waiting for and waited for takes time $O(|C| \cdot m)$.
- In general, we can observe d components at a time in order to minimize the error in our reachability analyses.

What is it good for? - A Trilateration System



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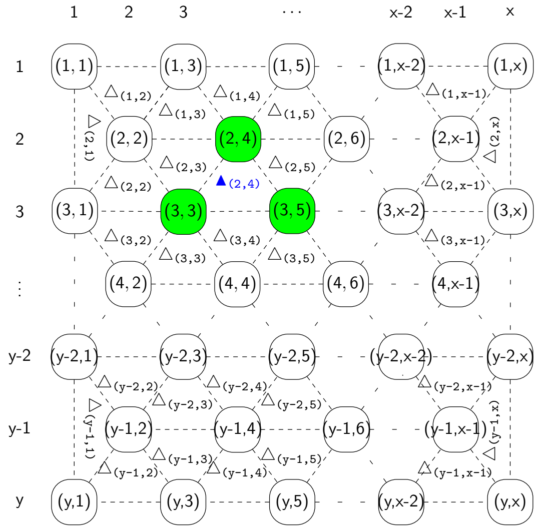
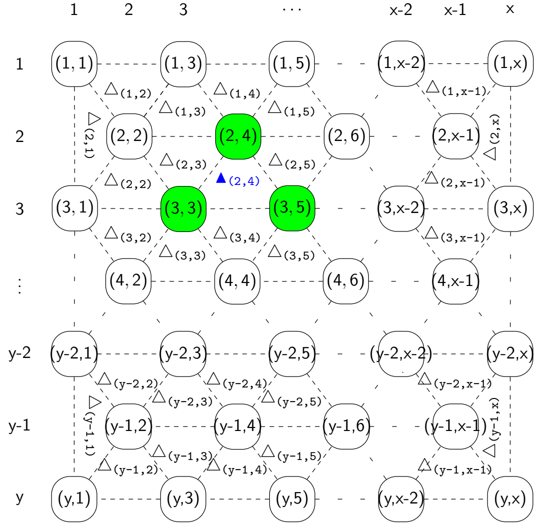


Three components that constitute a triangle may start, perform and end a trilateration cooperation.

What is it good for? - A Trilateration System



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Each component may participate in a communication of one of its surrounding triangles at a time.

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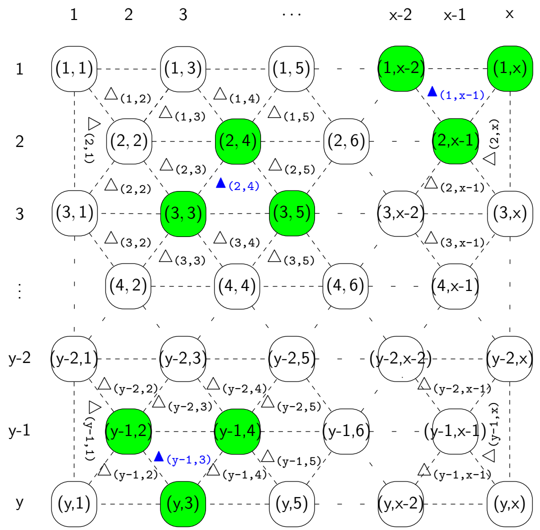

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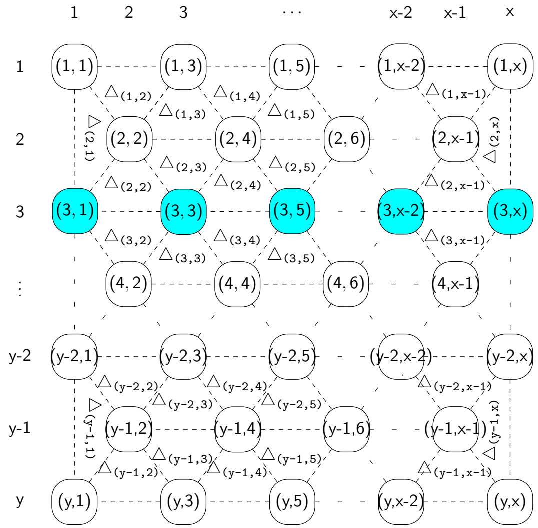

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⇒ Arbitrarily large connectors.

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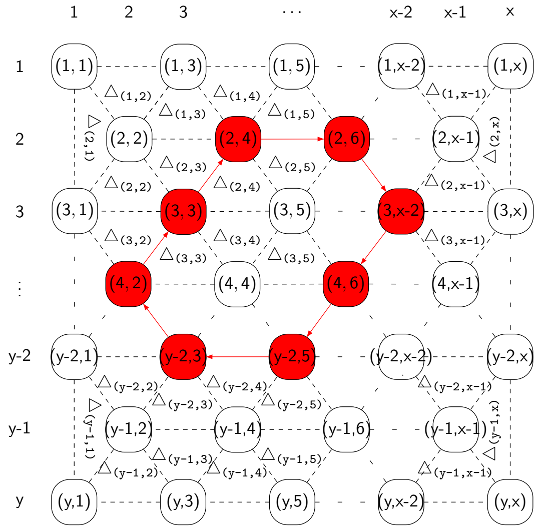

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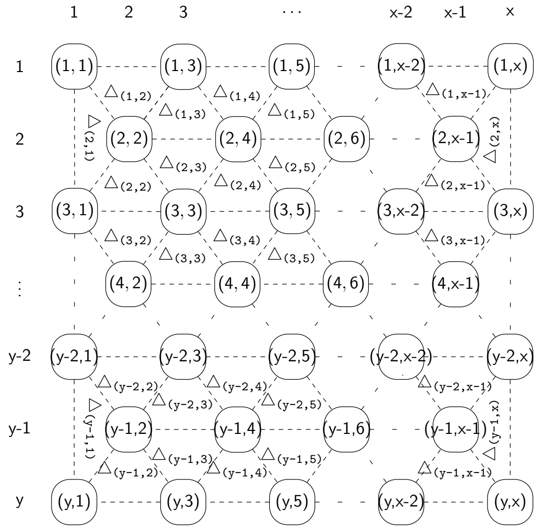

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The system can be proven deadlock-free by observing subsystems of size 3! √

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- The size of the subsystems serves as a parameter which enables us to do a trade-off between time and accuracy