About the Termination Detection in the Asynchronous Message Passing Model

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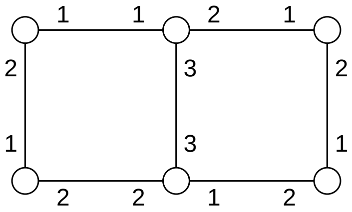
Message passing systems

A network is represented as a graph G with a port-numbering δ where each process can

- modify its state,
- send a message via port p,
- receive a message via port q.



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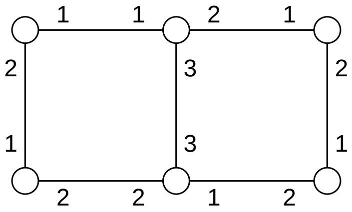
We consider

- reliable networks,
- (potentially) anonymous systems.
- asynchronous systems.



Motivations

Different kinds of termination exist for distributed algorithms.

- Implicit termination:
  - in the final configuration, each process has computed a correct result,
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Question

When can we transform a distributed algorithm with implicit termination into an algorithm with explicit termination ?

Motivations

What initial knowledge must we have about the network to enable this transformation ?

- the topology of the network,
- its size,
- a bound on its size,
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Equivalently, what are the families of graphs where we can transform any implicitly terminating distributed algorithm into an explicitly terminating algorithm ?

“Naive” strategy

- compute a snapshot,
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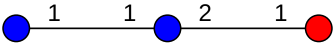
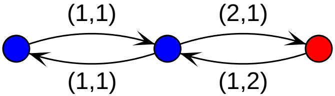
Problem

This ideal strategy cannot always be successfully applied.

From graphs to digraphs

We represent the network by a labelled digraph (G, δ, λ) where the state of each process is encoded by its label.

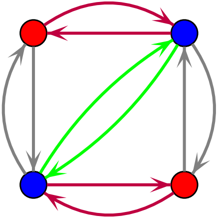




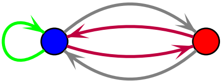
Coverings

Definition

D is a covering of D' via φ if φ is a locally bijective homomorphism.



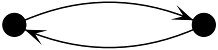
D

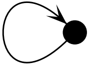


D'

Lifting Lemma

(G, δ_G)





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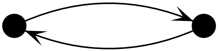

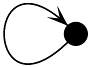

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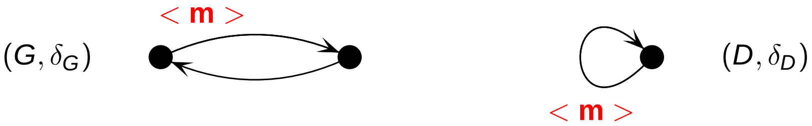

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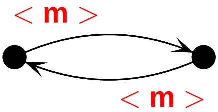

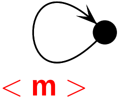

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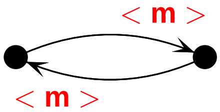

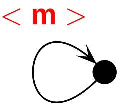

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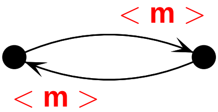

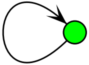

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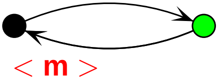

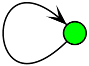

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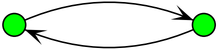

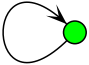

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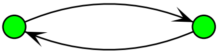

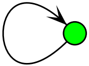

(D, δ_D)

Lemma (from Angluin'80)

$$
\begin{array}{ccc}
(G, \lambda, \delta_G) & & \\
\text{covering} \downarrow & & \\
(D, \eta, \delta_D) & \xrightarrow[\mathcal{A}]{} & (D, \eta', \delta_D)
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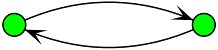

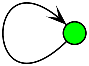

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A “better” strategy

- compute a “snapshot” up to a covering,
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  - no process can modify its state (according to the algorithm),
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An algorithm to construct coverings

Proposition (from Mazurkiewicz'97)

There exists an algorithm $\mathcal{M}$ that terminates implicitly on any graph (G, λ, δ_G) that computes a graph (D, η, δ_D) such that:

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One can obtain an algorithm $\mathcal{M}(\mathcal{A})$ that terminates implicitly on any graph $(G, \lambda, \delta) \in \mathcal{F}$ and that yields to a final labelling $(G, (\lambda, res_{\mathcal{A}}, res_{\mathcal{M}}), \delta)$ such that :

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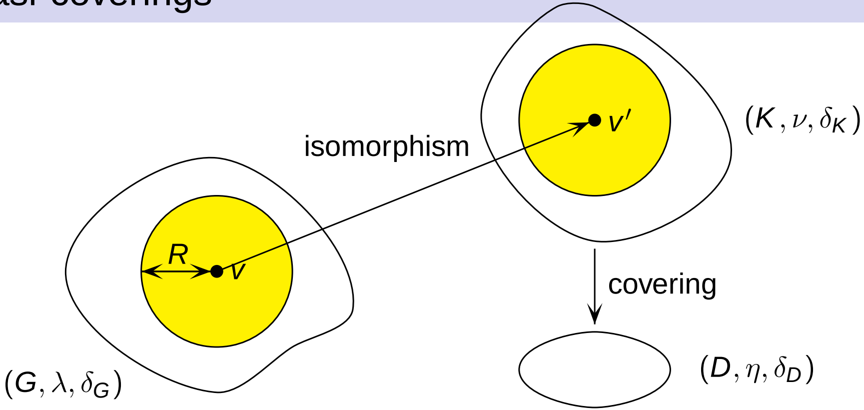
What is the meaning of the states of the processes during the execution of our “snapshot” algorithm ?

Problems

In the litterature, some impossibility results exist

- Angluin '80 : about the detection of the termination.
- Métivier, Muscholl, Wacrenier '97 : introduction of quasi-coverings.

Quasi-coverings



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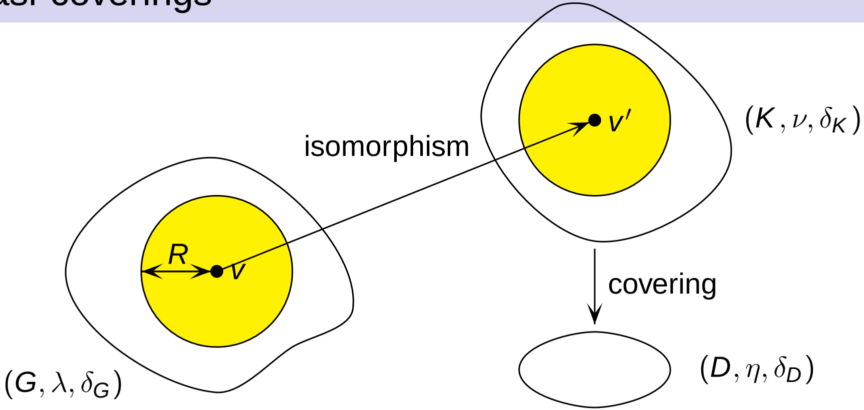
Quasi-coverings



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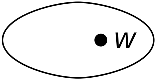
If $V(G) \setminus B_G(v, k) \neq \emptyset$, the quasi-covering is strict.

Impossibility result (from MMW'97)

$(D, \eta, \delta_D) \in \hat{\mathcal{F}}$



- Let $\hat{\mathcal{F}} = \{(D, \eta, \delta_D) \mid \exists (G, \lambda, \delta_G)$ that covers $(D, \eta, \delta_D)\}$.
- Consider $\mathcal{T}$ that detects termination on $\mathcal{F}$ and a synchronous execution ρ of $\mathcal{T}$ on (D, η, δ_D).

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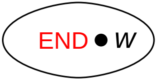

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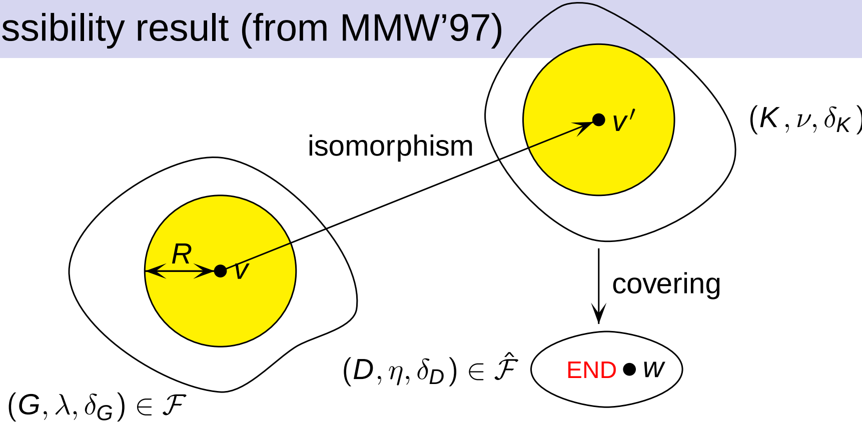

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- Consider $\mathcal{T}$ that detects termination on $\mathcal{F}$ and a synchronous execution ρ of $\mathcal{T}$ on (D, η, δ_D).
- Let r be the number of rounds of ρ.
- Suppose there exists $(G, \lambda, \delta_G) \in \mathcal{F}$ that is a strict quasi-covering of (D, η, δ_D) of radius $R > r$.

Main result

Theorem

Given a recursive family $\mathcal{F}$ of networks, one can detect the termination of $\mathcal{A}$ on $\mathcal{F}$

$$\Longleftrightarrow$$

there exists a computable function $r : \hat{\mathcal{F}} \to \mathbb{N}$ such that for any $(D, \lambda, \delta) \in \hat{\mathcal{F}}$, there is no strict quasi-covering of (D, λ, δ) of radius $r(D, \lambda, \delta)$ in $\mathcal{F}$.

A “good” strategy

- compute a “snapshot” up to a quasi-covering,
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Detecting the termination of $\mathcal{M}(\mathcal{A})$

- During the computation of $\mathcal{M}(\mathcal{A})$ on (G, λ, δ_G), each vertex knows a graph $(D, \eta, \delta_D) \in \hat{\mathcal{F}}$ such that (G, λ, δ_G) is a quasi-covering of (D, η, δ_D).

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- If $R > r(D, \eta, \delta_D)$, then the quasi-covering cannot be strict (by definition of the function r).
- (G, λ, δ_G) is a covering of (D, η, δ_D) and all vertices of G have computed their final values.

Applications

- Known corollaries :
  One can detect termination of an algorithm $\mathcal{A}$ on a network (G, λ, δ) if one of the following conditions is satisfied.
  - existence of a leader,
  - unique ids,
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- New corollaries :
  One can detect termination of an algorithm $\mathcal{A}$ on a network (G, λ, δ) if one of the following conditions is satisfied.
  - existence of at most k distinguished vertices,
  - initial knowledge of a bound on the multiplicity of each initial label.