Maximum finding in the symmetric radio networks with collision detection

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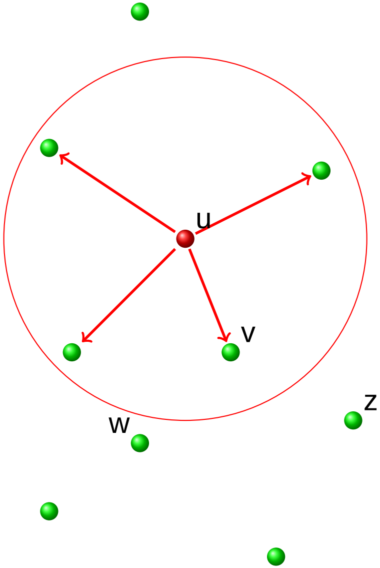
What is radio network ?

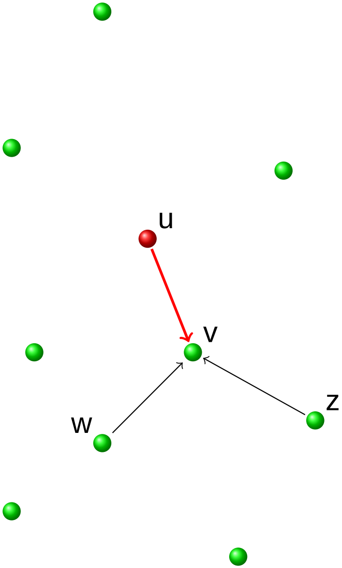
- a collection of receiver-transmitter devices - nodes
- nodes are autonomous
- communication via sending messages
- **single** shared communication frequency
- nodes work in globally synchronised time slots - rounds
- in each round, each node makes a decision:
  act either as a receiver or as a transmitter
- multihop network - modelled by a reachability graph

Communication in radio networks



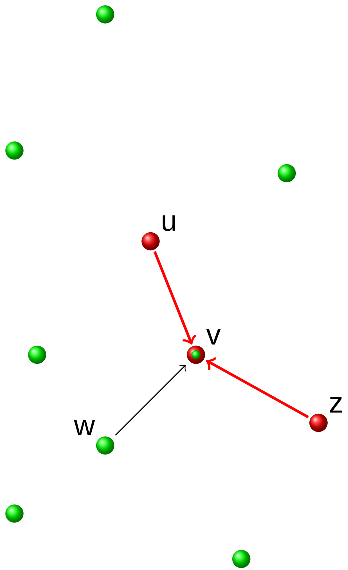
- if a node *u* transmits, then the signal from *u* goes to all nodes within its transmission range

Communication in radio networks



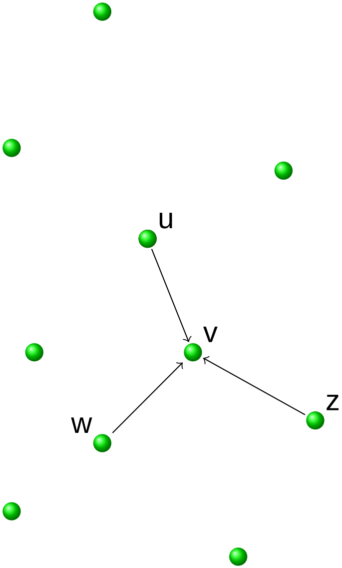
- if a node u transmits, then the signal from u goes to all nodes within its transmission range
- if the node v listen: then it receives message from u if and only if u is the only transmitting node which has v in its transmission range

Communication in radio networks



- if a node *u* transmits, then the signal from *u* goes to all nodes within its transmission range
- if the node *v* listen: and is in the range of more than one transmitting node, then collision occurs and no message is received - the node *v* hears **interference noise**

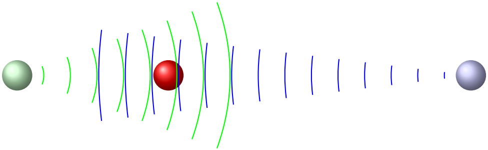
Communication in radio networks



- if the node v listen:
  and v is in the range of
  no transmitting node,
  then v hears
  background noise

Key features of radio communication

- interference of simultaneous transmissions (slowdown)
  - occurs if a node is in the range of more than one transmitting node



- omnidirectional transmission (speedup)
  - a node transmits to all nodes located within its transmission range

Reachability graph, measure of effectiveness

Reachability graph of a radio network is a directed graph $G = (V, E)$, where

- vertex set V corresponds to the nodes of the network
- an edge $e = (u, v) \in E \iff$ the node v is in the transmission range of the node u

Measure of effectiveness

- mostly the **time** required **to complete** the prescribed communication task
- other studied measures: energy consumption, combined measures, etc.
- measured according to parameters of reachability graph (diameter D, eccentricity *ecc* of the source, number of nodes n, etc.)

Communication tasks

- broadcasting - the goal is to deliver a message from a distinguished node (source) to all nodes of the network
- gossiping - all nodes have a message, the goal is to distribute each message to all nodes of the network
- maximum finding - each node possesses a (integer) value, the goal is to compute the maximal possessed value in a distinguished node (initiator)
- ...

Different communication scenarios

- directed or undirected (symmetric) network
- known network, unknown network or partially known network
- no node labels (anonymous nodes)
  nodes with labels $(1, \ldots, O(n))$
- randomized or deterministic protocol
- bounded or unbounded messages
- collision detection or no collision detection

Collision detection

- a **listening node** is able to distinguish **interference noise** (more than one transmitting neighbour) and **background noise** (no transmitting neighbour)

Effectiveness is significantly influenced by considered model.

Our setting

Model of radio network:

- undirected (symmetric) network (e.g. transmission power of all nodes is the same)
- unknown network
- no node labels (anonymous nodes)
- deterministic protocol, bounded messages
- collision detection capability

Problem: **Integer maximum finding**

- each node possesses **an integer value**
- a distinguished node (initiator) has the goal to **compute** the **maximal** possessed value
- the initiator starts in the time (round) **unknown** to other nodes

Our setting - example of motivation

- sensor devices measuring a physical quantity and communicating via low-power radio
- one central node (operator) needs to known the maximal measured value (e.g. maximal radiation)
- limited precision of real measurement - integer values

Main result

Theorem

There is an asymptotically optimal algorithm computing the value Max (in the initiator) in $\Theta(ecc + \log Max)$ rounds, where

- *Max is the maximal integer value over possessed values*
- *ecc is the eccentricity of the initiator, i.e. maximal distance from the initiator to any other node of the network.*

Used techniques

- encoding information into collisions
- pipelining (wave technique)
- utilised properties of binary encoding of integers

Subroutine algorithm RBEM

- pipelined broadcast algorithm using encoding information into collisions
- broadcasts message M of binary length m in $O(ecc + m)$, where ecc is the eccentricity of the source node
- presented by Okuwa, Chen, Wada (2003)

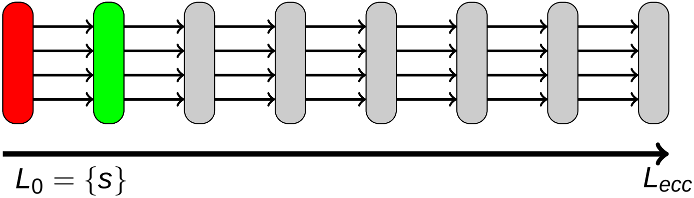
Encoding information into collisions (1)

- a distinguished message - contact message
- **encoding** of 1 bit information:
  - ”0” . . . no message is transmitted
  - ”1” . . . contact message is transmitted
- **decoding** of the received information (listening node):
  - no message received
    $\Longrightarrow$ all neighbours (if exist) said ”0”
  - contact message received or collision detected
    $\Longrightarrow$ at least one neighbour said ”1”

Encoding information into collisions (2) - example

- Layer - a set of nodes in the same distance from a distinguished node (e.g. initiator, source)
- **simple broadcast algorithm**: message transformed to
  - opening sequence (e.g. 11)
  - message content not including the closing sequence (e.g. 1 $\rightarrow$ 01, 0 $\rightarrow$ 00)
  - closing sequence (e.g. 11)

  and distributed layer by layer
- only two neighbouring layers work in the same time



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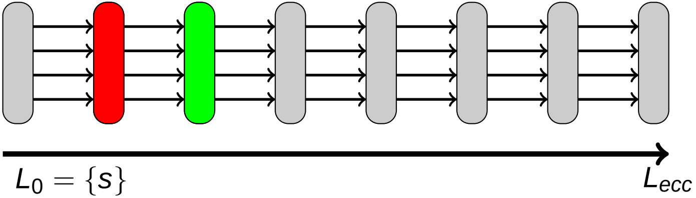

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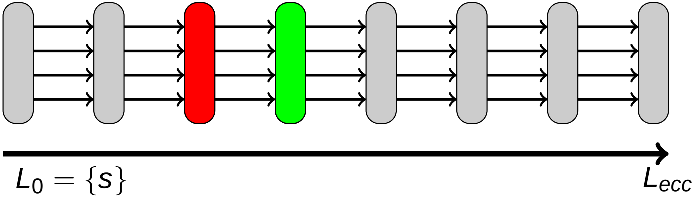

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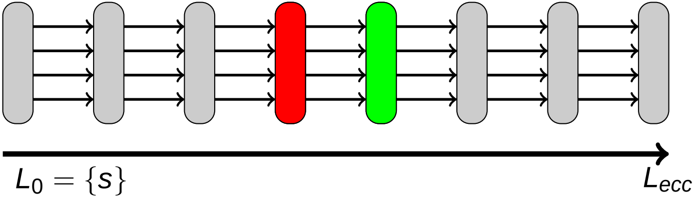
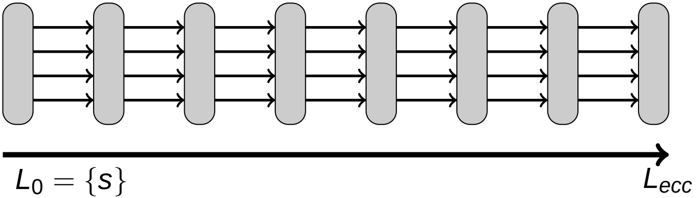


Pipelining (wave technique)

- solution for **"simultaneous" communication** in all layers **without "interference"**
- pipelined broadcasting: immediately after recognizing next bit of message, it is sent to the next layer
- layers **divided into 3 groups** (distance modulo 3), cyclic changing of action
  - transmit segment
  - receive segment
  - sleep segment - separation function



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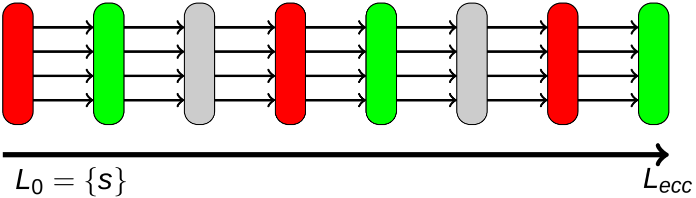

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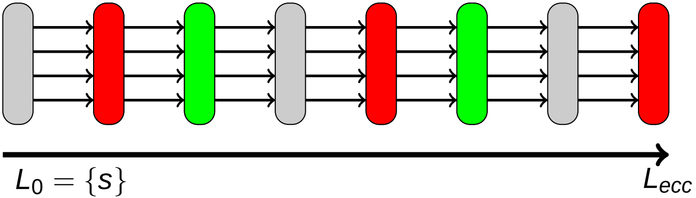

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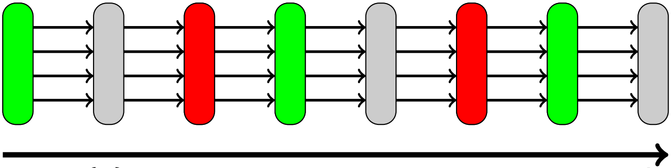


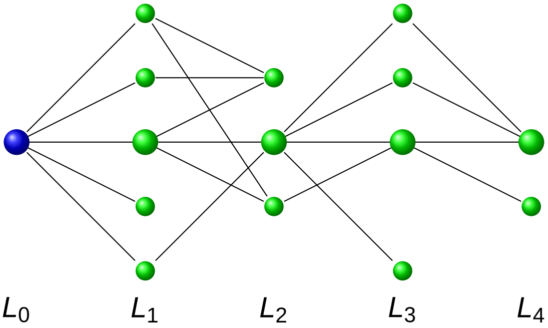
$L_0 = \{s\}$ L_{ecc}

Rough description of algorithm

- Preprocessing
  1. in the initiator, compute eccentricity of the initiator
     $O(ecc)$ rounds
  2. broadcast computed eccentricity *ecc*
     $O(ecc)$ round using *RBEM* algorithm
  3. in each node computes its distance from the initiator
     $O(ecc)$ rounds
     side effect: **synchronisation of nodes**
- Computation of the maximal value
  1. in the initiator, compute estimation B_{max} of maximal value Max such that $2^{B_{max}-1} \leq Max < 2^{B_{max}}$
     $O(ecc + \log Max)$ rounds
  2. broadcast computed estimation B_{max}
     $O(ecc + \log Max)$ rounds using *RBEM* algorithm
  3. in the initiator, compute the value *Max*
     $O(ecc + \log Max)$ rounds

Eccentricity computing (1)

- works in 2-part phases:
- 1^{st} part activates nodes in the next layer and provides information about modulo 3 distance (4 rounds)
- 2^{nd} part informs active nodes about existence of their active $+1$-neighbours (6 rounds)



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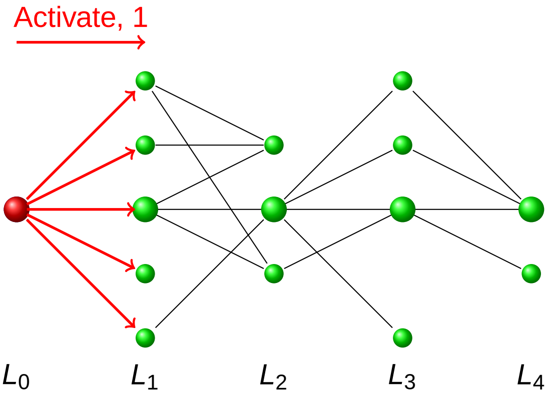

Part: 1

Phase: 1

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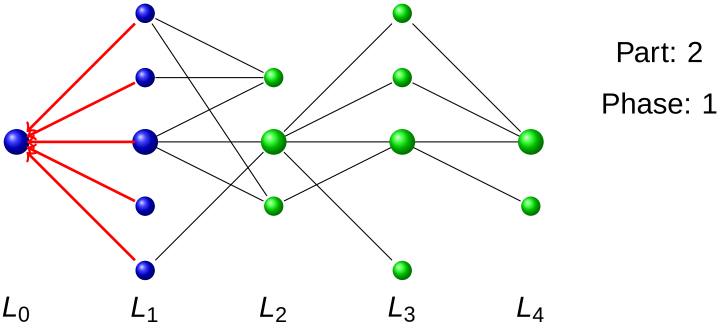

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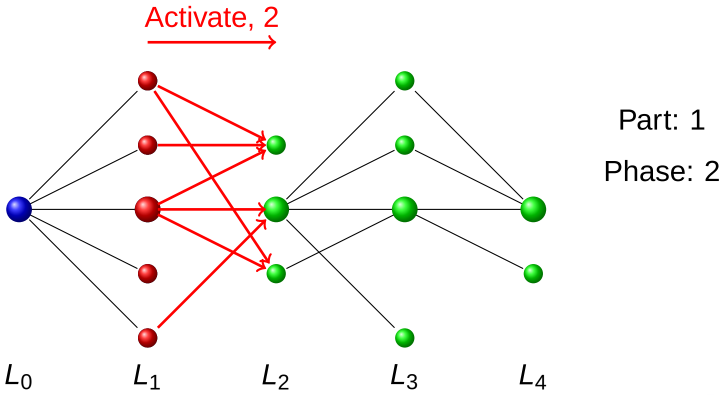

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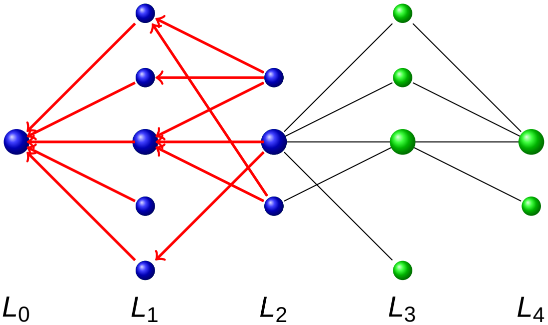

Part: 2

Phase: 2

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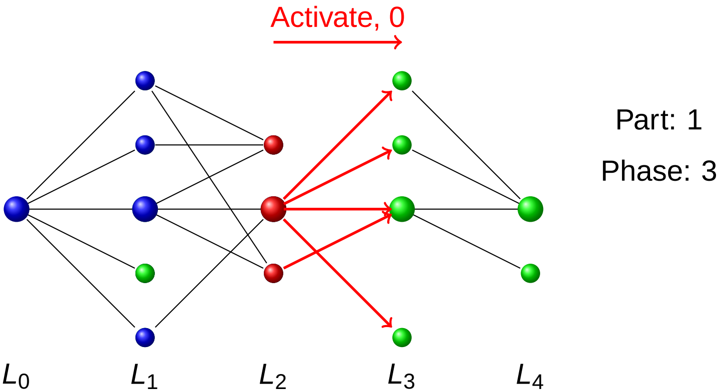

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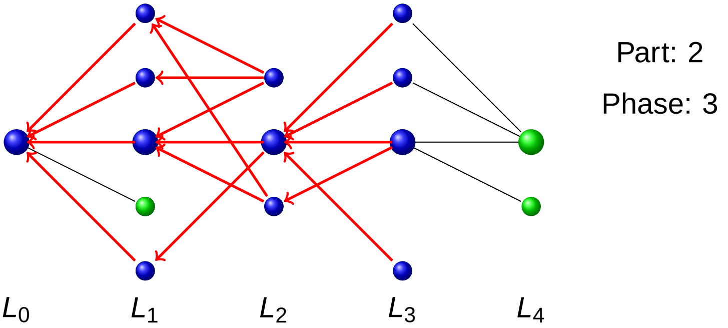

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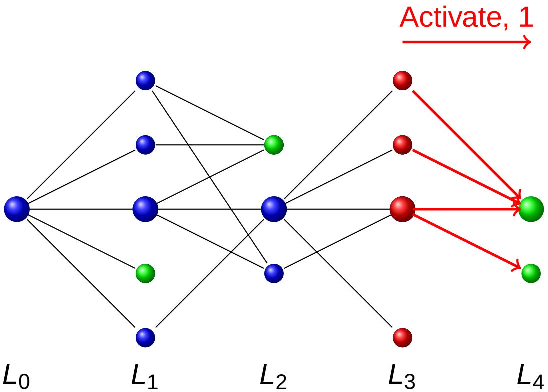

Part: 1

Phase: 4

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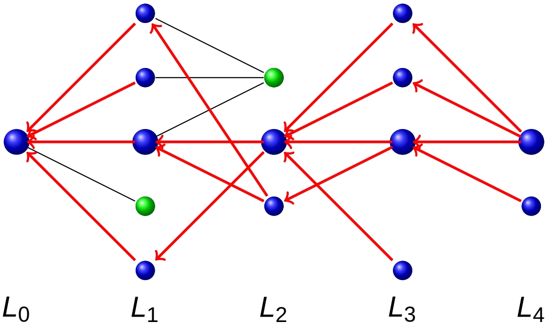

Part: 2

Phase: 4

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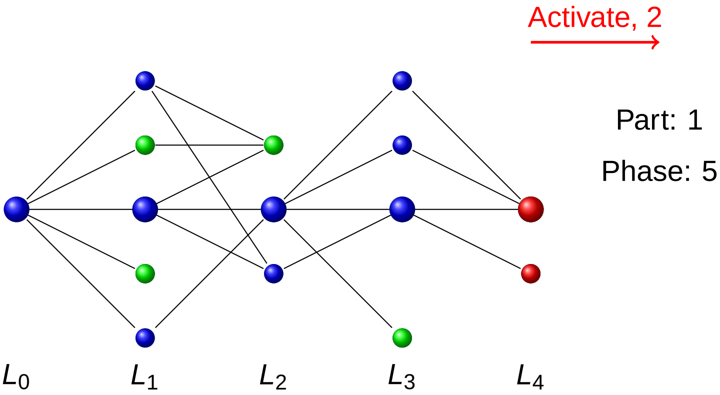

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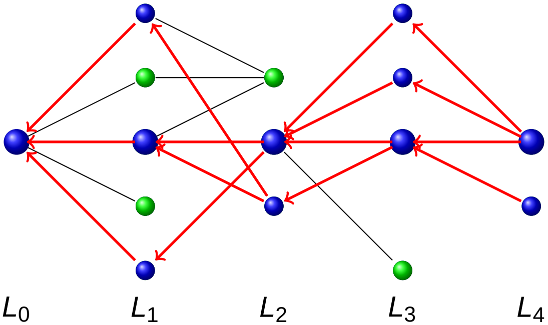

Part: 2

Phase: 5

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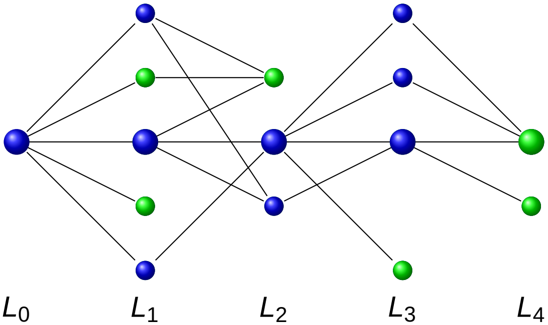

Part: 1

Phase: 6

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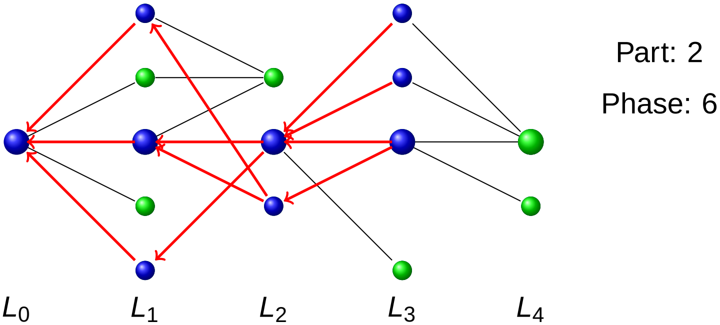

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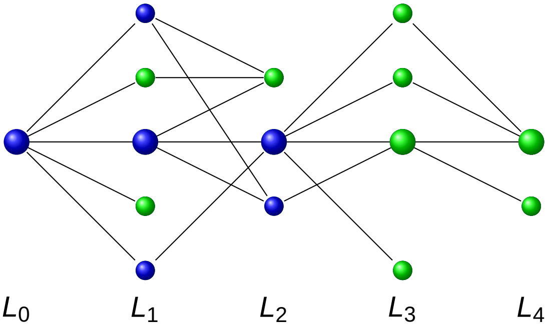

Part: 1

Phase: 7

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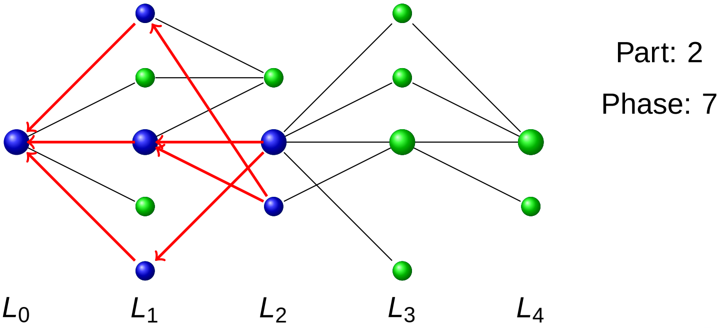

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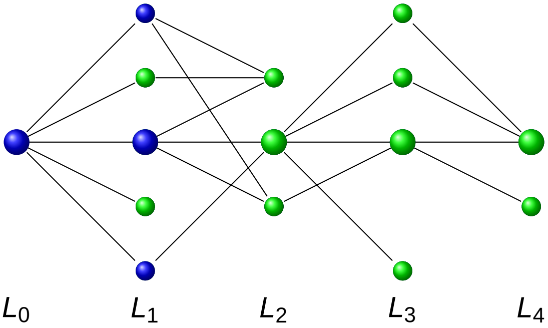

Part: 1

Phase: 8

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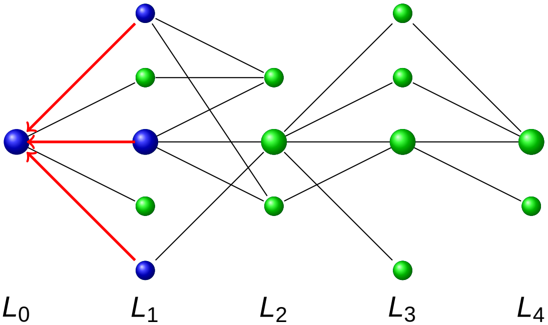

Part: 2

Phase: 8

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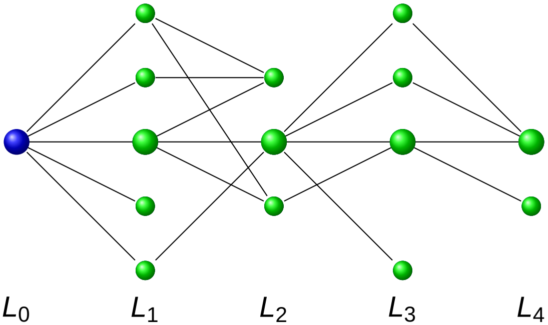

Part: 1

Phase: 9

Eccentricity computing (1)

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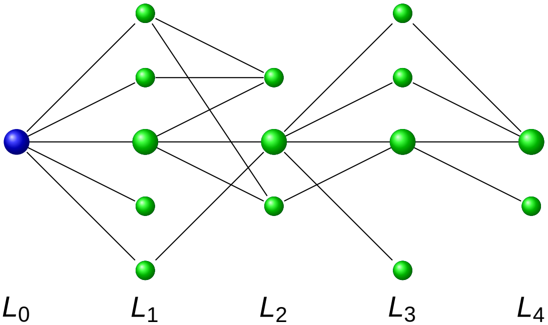

Part: 2

Phase: 9

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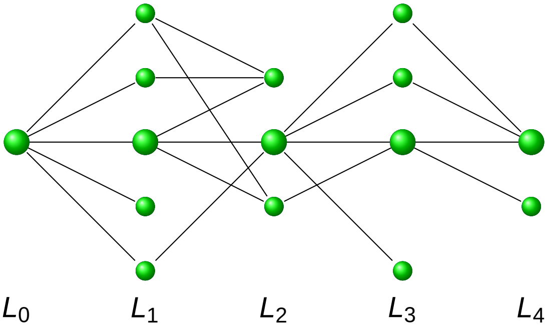


Part: 1

Phase: 10

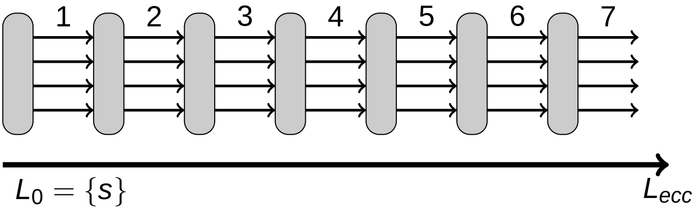
$10 = 2.4 + 2$

Eccentricity computing (2) - technical details

- first part uses special **opening sequence** and encodes **modulo 3 distance** by appropriate transmissions
- opening sequence **sets up counter** of the actual round of the current phase
- second part is scheduled using known modulo 3 distances of active nodes and **avoiding** transmission of **opening sequence** of the first part
- $C = 2.ecc + 2$, where C is the number of phases in which the initiator is active

Distance computing

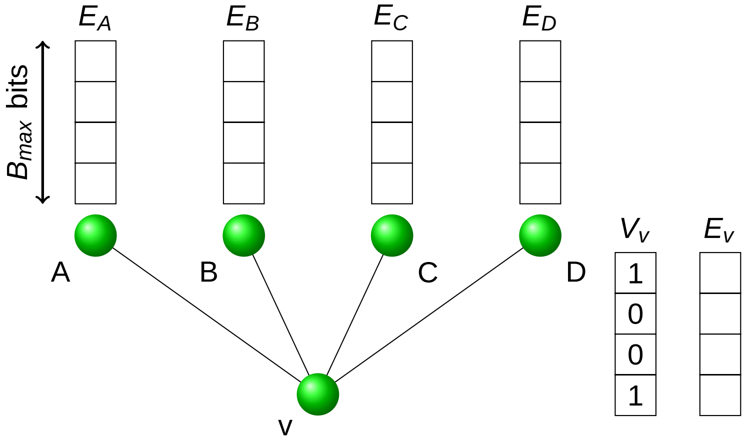
- simple modification of *RBEM* (pipelined broadcast) algorithm
- nodes of the layer L_i **dynamically change** broadcasted message to the binary encoded number $i+1$
- based on the fact, that knowing k lowest bits of a number j we know k lowest bits of the number $j+1$



Maximum computing (1)

- suppose, that the value B_{max} is known by all nodes
  - $2^{B_{max}-1} \leq Max < 2^{B_{max}}$
  - algorithm for computing B_{max} is based on simplified idea of the maximum computing algorithm
  - B_{max} broadcasted by *RBEM*
- all values V_v (possessed integer values) can be considered as **binary sequences** of the **same length** B_{max}
- each node computes E_v - its **estimation** of the maximal value
- node is **active** only if its estimation can be **"useful" in the following computation** - otherwise it is deactivated

Maximum computing (2) - example



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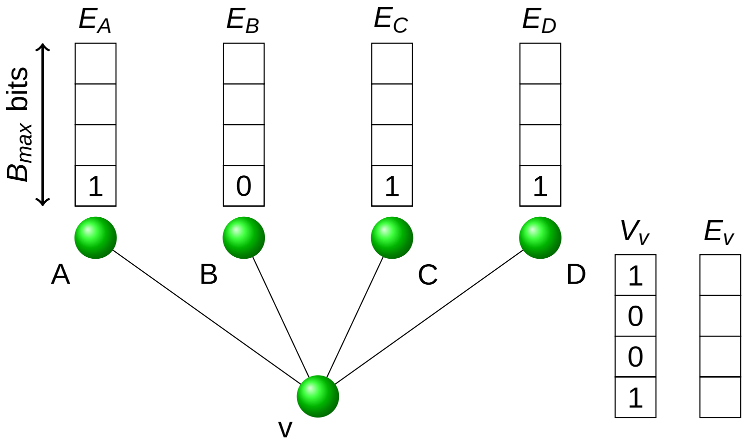
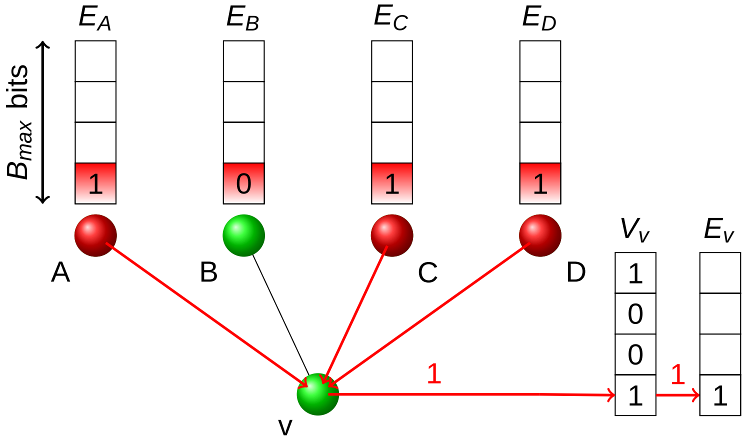


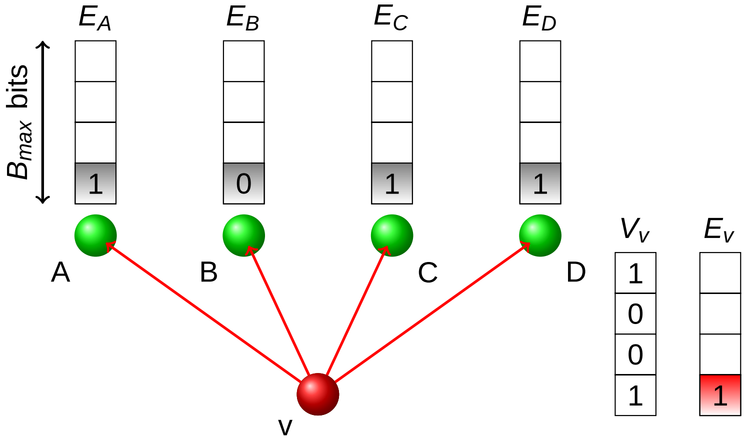
$+1$-neighbours of v have updated their estimations

Maximum computing (2) - example



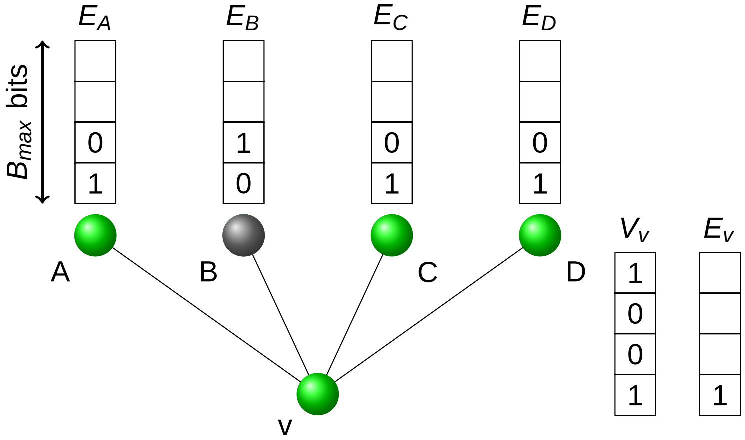
$+$1-neighbours inform v about active bits of their estimations

Maximum computing (2) - example



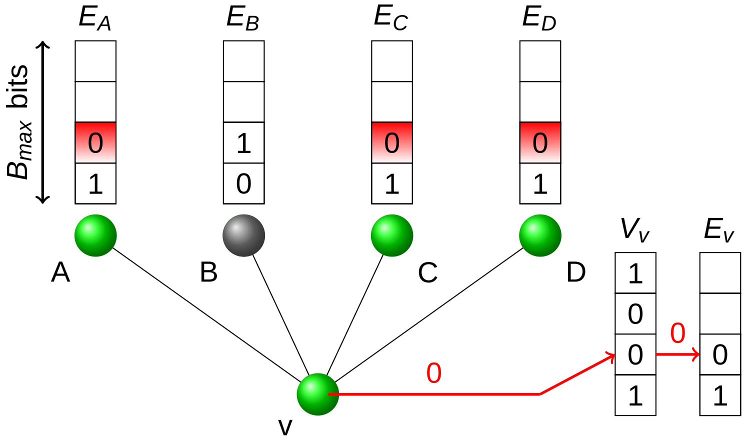
v informs $+1$-neighbours about its setting of active bit

Maximum computing (2) - example



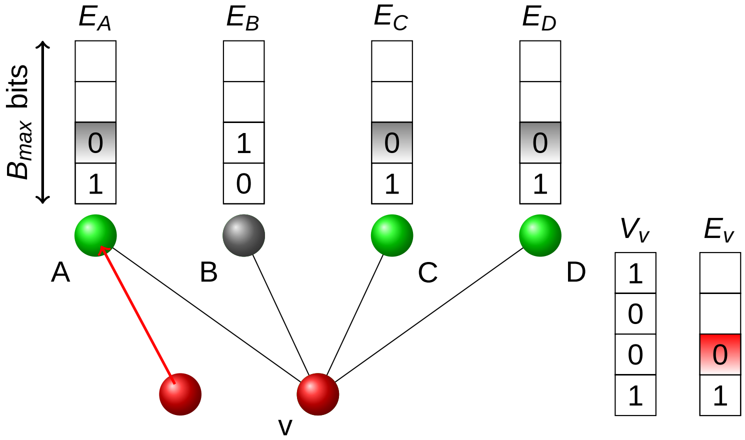
B is deactivated - it cannot help

Maximum computing (2) - example



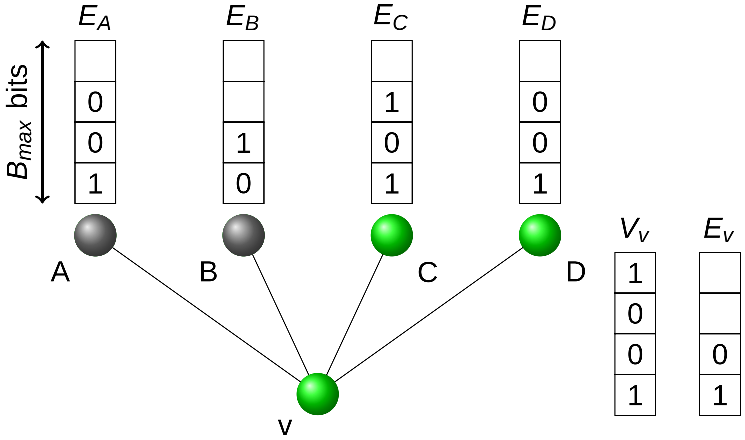
$+1$-neighbours inform v about active bits of their estimations

Maximum computing (2) - example



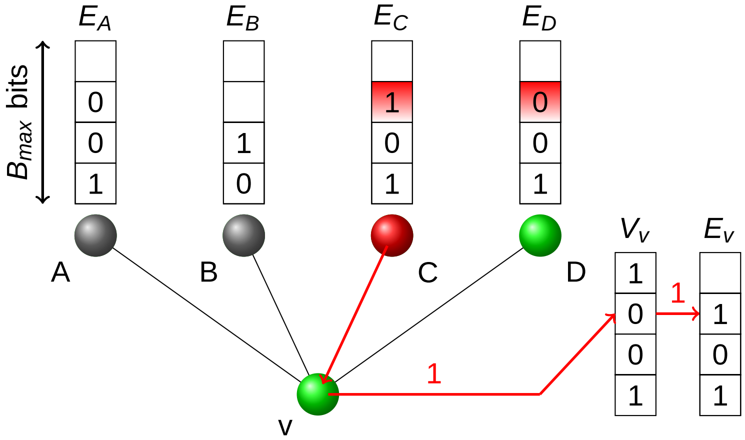
A can have other -1-neighbours ...

Maximum computing (2) - example



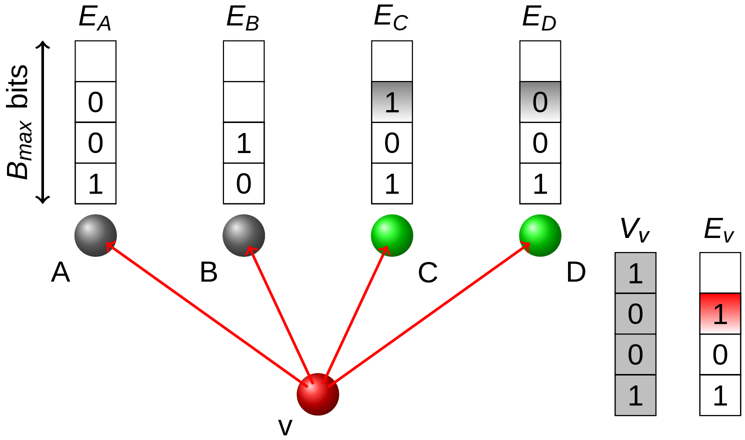
A is deactivated - stopped by a -1 neighbour

Maximum computing (2) - example



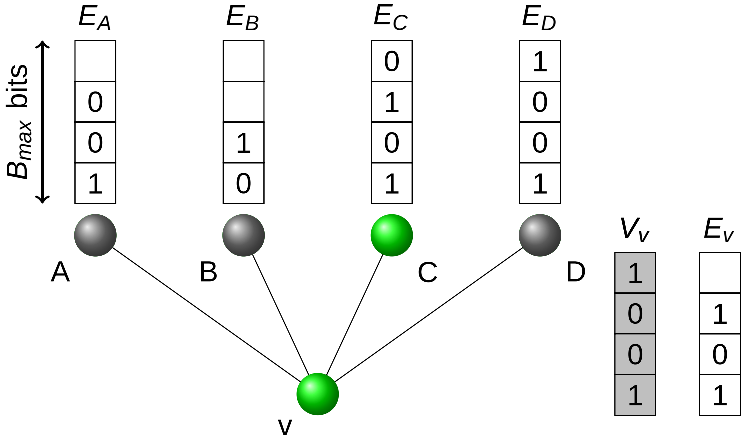
$+1$-neighbours inform v about active bits of their estimations

Maximum computing (2) - example



value V_v is too small and will be never considered

Maximum computing (2) - example



D is deactivated . . .

Maximum computing (3)

- transmissions are scheduled according to the (known) distances from the initiator
- E_v is always "greater" than V_v and "less" than all reachable estimations in higher layers
- "maximal value" always reaches initiator
- if E_s is fully computed, it is equal to the searched maximum

Total time of computation: $O(ecc + \log Max)$ rounds

Lower bound

Theorem

For any maximum finding algorithm with collision detection there exists a symmetric radio network of diameter 2 and such an assignment of values associated to nodes that the algorithm requires $\Omega(\log Max)$ rounds.

- proof based on the result from Dessmark and Pelc: $\Omega(\log n)$ lower bound of broadcasting in symmetric geometric radio networks (GRN) with collision detection

Therefore designed algorithm is asymptotically optimal.

Conclusion

- we design $\Theta(ecc + \log Max)$ maximum finding algorithm for anonymous symmetric radio networks with collision detection
- **possible utilisations**:
  - maximum finding algorithm in the case of ”real” (binary encoded in the mantissa/exponent form) possessed values
  - computing unknown parameters of the network
  - ”synchronisation” of nodes
  - selection of particular specific node, e.g. node with maximal label
  - utilisation for broadcasting and gossiping (already utilised for broadcasting in planar graphs)
- future research:
  - networks without collision detection

Thank you for attention