Improved upper bounds for λ-backbone colorings along matchings and stars

Bert Marchal
joint work with:

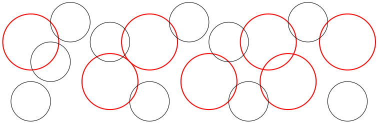
Hajo Broersma, Daniel Paulusma, A.N.M. Salman

January 23, 2007

frequency assignment

setting

- network of transmitters
- interference if they are close and broadcast on the *same* frequency
- stronger transmitters also interfere if they broadcast on *similar* frequencies

set of transmitters 

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Assign frequency channels to the transmitters in such a way that interference is avoided and bandwidth is minimized.

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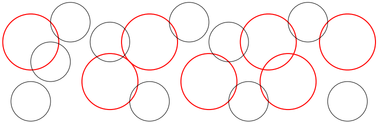

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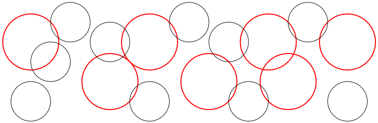

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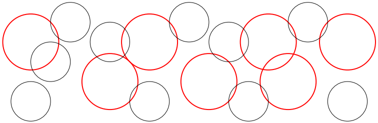

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general model

graph representation

- vertices of G represent transmitters
- vertices are adjacent when corresponding transmitters are likely to interfere
- edges between stronger transmitters that interfere when they broadcast on *similar* frequencies form a subgraph of G

general framework

- graph G_1 and subgraph G_2 of G_1
- determine coloring satisfying restrictions of type 1 in G_1 and of type 2 in G_2 using minimum number of colors

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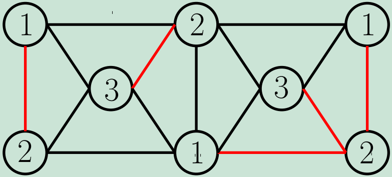
examples

example 1

G_2 (red edges) is a subgraph of G_1



type 1: adjacent vertices have different color numbers
type 2: adjacent vertices have different color numbers

example 2

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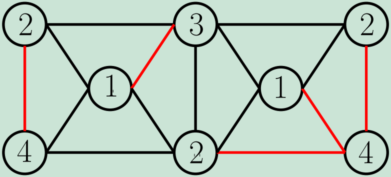

type 1: adjacent vertices have different color numbers
type 2: adjacent vertices have color numbers that differ by at least 2

definitions

backbone

- backbone of network is substructure of transmitters that are more crucial than the rest
- denoted by H and forms a spanning subgraph of the network graph G

λ-backbone coloring of (G, H)

- neighbors in H have color numbers that differ by at least λ ($\lambda \geq 2$)

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- star S_q is complete 2-partite graph with root r and q leaves
- star backbone is spanning collection of pairwise disjoint stars

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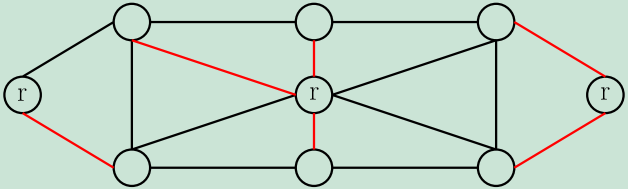

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- application: e.g. sensor networks

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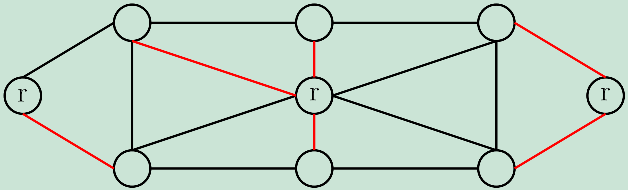

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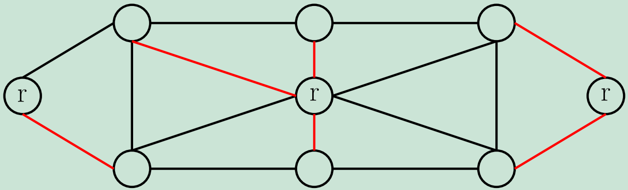

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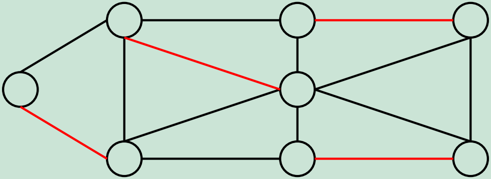

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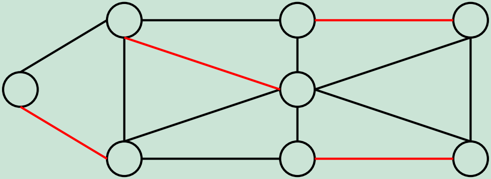

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previous results

Broersma et al. (2004)

- ub for $BBC_\lambda(G, M)$ roughly grow like $(2 - \frac{1}{\lambda})\chi(G)$
- ub for $BBC_\lambda(G, S)$ roughly grow like $(2 - \frac{2}{\lambda+1})\chi(G)$
- multiplicative factor (bad!)

Broersma, Fomin, Golovach and Woeginger (2003)

- for *split graphs* G and tree backbones T, $BBC_2(G, T) \leq \chi(G) + 2$

idea

additive constant (depending on λ) in split graphs ?

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split graphs

definition

A split graph is a graph whose vertex set can be partitioned into a *clique* and an *independent set* with possibly edges in between.

motivation

1. nice structural properties
2. every graph can be turned into a split graph

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upper bounds on $BBC_\lambda(G,S)$

Theorem

Let $\lambda \geq 2$ and let G be a split graph with $\chi(G) = k \geq 2$. For every star backbone S of G,

$$BBC_\lambda(G,S) \leq \begin{cases} k+\lambda & \text{if } k=3 \text{ and } \lambda \geq 2 \text{ or } k \geq 4,\ \lambda = 2 \\ k+\lambda-1 & \text{in the other cases.} \end{cases}$$

The bounds are tight.

$\Rightarrow$ additive constant depending on λ

example

split graph G and star backbone S with $\chi(G) = 5$ and $\lambda = 2 \Rightarrow$ $BBC_2(G,S) \leq 7$

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old and new upper bounds compared

previous results:

- for any graph G with matching or star backbone
- upper bounds are roughly a factor times $\chi(G)$

new results:

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- consider split graph $G = (V, E)$ with matching backbone M
- partition V in a largest clique C and an independent set I
- let $nn(v)$ denote the set of non-neighbors of vertex v
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definition

A splitting set S is a subset of I such that $\bigcup_{v \in S} nn(v)$ does not intersect with $\bigcup_{v \in S} mn(v)$.

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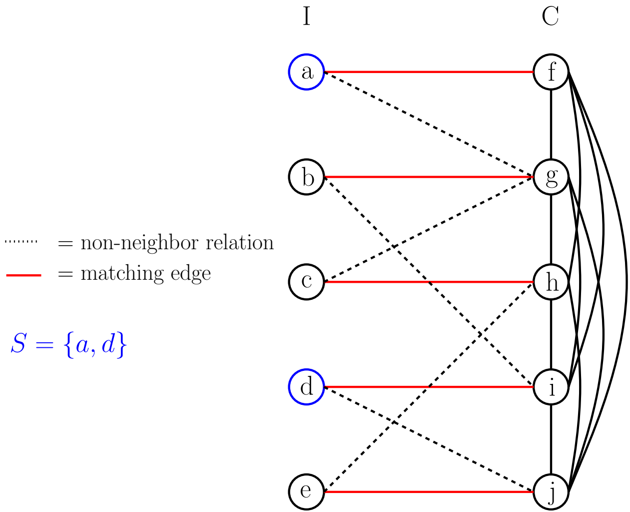

$S = \{a, d\}$

splitting set lemma

Lemma

Given (G, M), let $k = \omega(G) = |C|$ and let $i = |I|$. If every vertex in I has exactly one non-neighbor in C and $\lceil \frac{k}{3} \rceil \geq p$, then (G, M) has a splitting set S with $|S| = p - \frac{k-i}{2}$ such that there are no matching edges between elements of the set of non-neighbors of vertices of S.

constructive proof!

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matching case: $k = 10$, $\lambda = 5$

Thm: $BBC_5(G, M) \leq 11$

Lemma: $S = \{a, f\}$

partition of C:

C_1: mn in C, nn in S

C_2: mn in I, nn in S

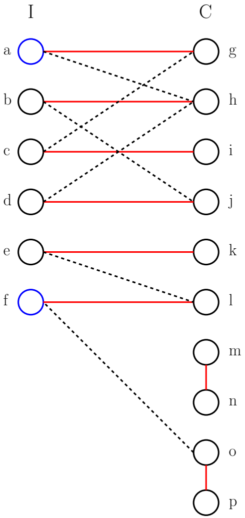
C_3: one ev of each me in C that has no ev in C_1

C_4: mn in I, no mn or nn in S

C_5: mn in S

C_6: not in C_1 or C_3,mn in C

coloring algorithm:



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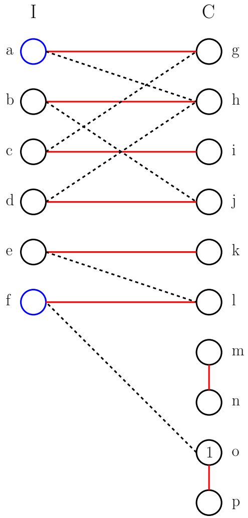
coloring algorithm:

1. $C_1 = \{o\}$ color with 1



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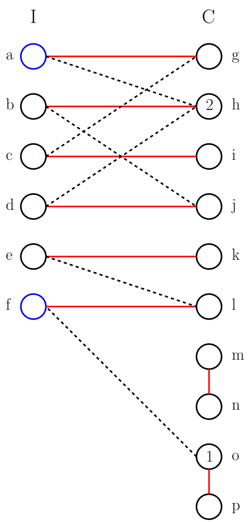
coloring algorithm:

1. $C_1 = \{o\}$ color with 1
2. $C_2 = \{h\}$ color with 2



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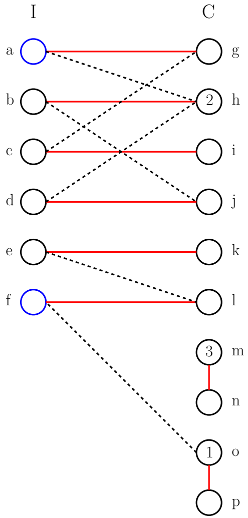
coloring algorithm:

1. $C_1 = \{o\}$ color with 1
2. $C_2 = \{h\}$ color with 2
3. $C_3 = \{m\}$ color with 3



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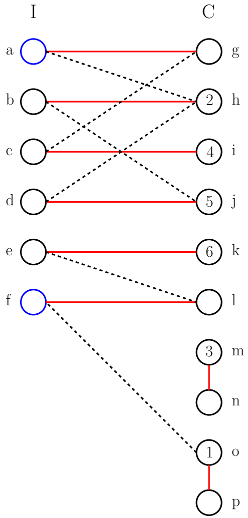
coloring algorithm:

1. $C_1 = \{o\}$ color with 1
2. $C_2 = \{h\}$ color with 2
3. $C_3 = \{m\}$ color with 3
4. $C_4 = \{i, j, k\}$, color with 4, 5 and 6



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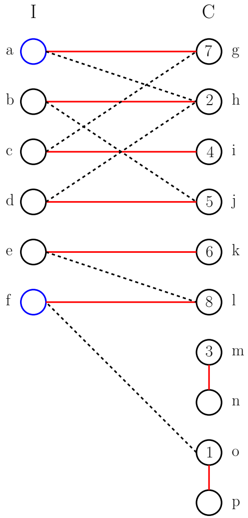
2. $C_2 = \{h\}$ color with 2
3. $C_3 = \{m\}$ color with 3
4. $C_4 = \{i, j, k\}$, color with 4, 5 and 6
5. $C_5 = \{g, l\}$, color with 7 and 8



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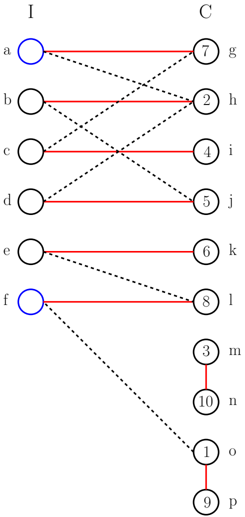
2. $C_2 = \{h\}$ color with 2
3. $C_3 = \{m\}$ color with 3
4. $C_4 = \{i, j, k\}$, color with 4, 5 and 6
5. $C_5 = \{g, l\}$, color with 7 and 8
6. $C_6 = \{n, p\}$, color with 9 and 10



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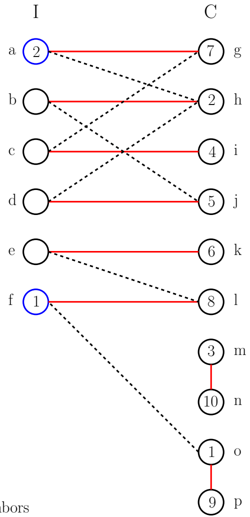
4. $C_4 = \{i, j, k\}$, color with 4, 5 and 6
5. $C_5 = \{g, l\}$, color with 7 and 8
6. $C_6 = \{n, p\}$, color with 9 and 10
7. color S by assigning same color as non-neighbors



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1. $C_1 = \{o\}$ color with 1
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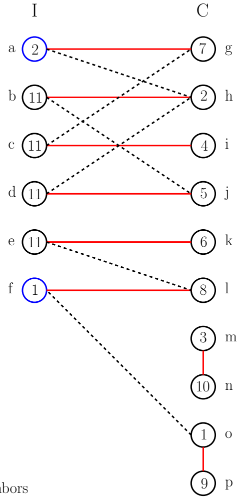
3. $C_3 = \{m\}$ color with 3
4. $C_4 = \{i, j, k\}$, color with 4, 5 and 6
5. $C_5 = \{g, l\}$, color with 7 and 8
6. $C_6 = \{n, p\}$, color with 9 and 10
7. color S by assigning same color as non-neighbors
8. color rest of I with 11



directions for further research

- obtain good upper bounds on $BBC_\lambda(G, T)$, where G is a split graph and T is a tree.
- obtain good upper bounds on $BBC_\lambda(G, H)$ for general G and H.

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