Adaptive Incentive-Compatible Sponsored Search Auction

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YAHOO! SEARCH flowers Search

Answers

Search Results 1 - 10 of about 158,000,000 for **flowers** - 0.21 sec. (About this page)

Also try: 1800 **flowers**, pictures of **flowers**, wedding **flowers**, spring **flowers** More...

SPONSOR RESULTS

- Send **Flowers** Everywhere in Belgium
  www.brusselsflowerexpress.com - Brussels **Flower** Express sends beautifull **flower** arrangements everywhere in Belgium.
- Netflores -The **Flower** Shop Brazil Trusts
  english.netflores.com.br - First Brazilian **flower** shop on the Internet, since 1995.
- Iris and Spring **Flowers**
  www.schreinersgardens.com - Free catalog. Watch your spring **flower** garden explode in a rainbow of color with our bearded iris.
- **Flowers** - Cheap Prices
  www.NexTag.com/flowersplants - **Flowers** & Plants by Occasion, Type & Color. **Flowers** on Sale.

Flowers - Image Results

More **flowers** images

Yahoo! Shortcut - About

1. 1-800-**FLOWERS**.COM - Official Site
   1-800-**FLOWERS**.COM offers a unique selection of gifts from **flowers** and plants to gourmet foods, chocolate, gift baskets and more. Same day nationwide delivery available.
   Quick Links: Spring - Same Day Delivery - Sympathy and Funerals
   www.1800**flowers**.com - More from this site
2. Proflowers Official Site - Fresh Cut **Flowers**, Roses, Plants and More
   The ProFlowers difference is changing the flower world, with fresh **flowers** from the field, fast delivery, and floral arrangements guaranteed to last 7 days.
   Quick Links: Best Sellers - Roses - Birthday **Flowers**
   www.pro**flowers**.com - More from this site
3. FTD.com
   Order & send fresh **flowers**, roses, bouquets, gift baskets & more at FTD - your online florist. We offer same day delivery on fresh **flowers** & plants.
   Quick Links: Roses - Gift Baskets - Best Sellers
   www.ftd.com - More from this site

SPONSOR RESULTS

FlowersandFreshness.com: Wedding **Flowers**
200 roses for $126.00 shipping included. Fresh **flowers** direct from...
www.flowersandfreshness.com

Whatcom Seed Co.: Uncommon **Flower** Seeds
A wide selection of quality **Flower** seeds for your home and garden,...
seedrack.com

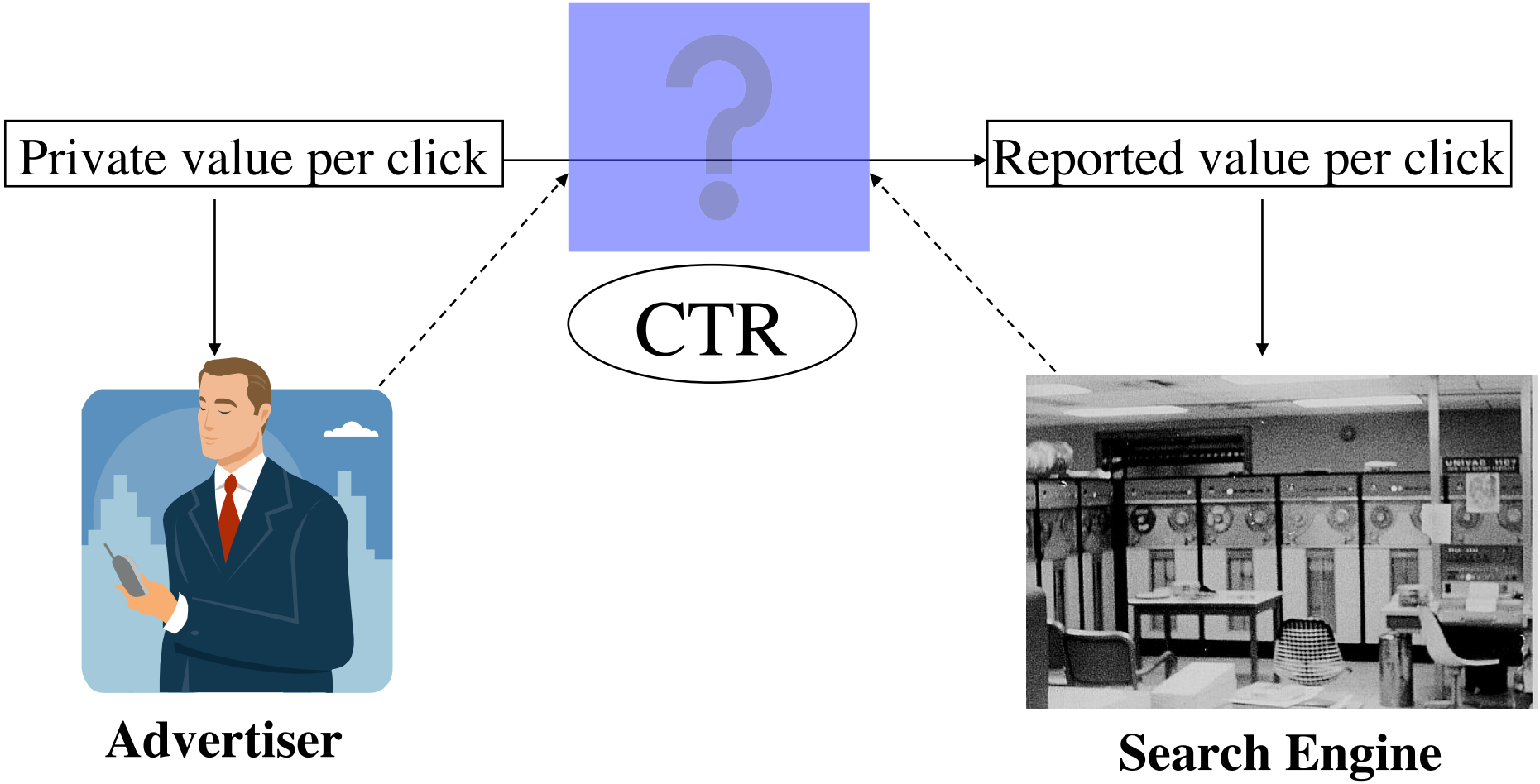
Garden **Flowers**
Find, compare & buy. Compare & Buy from 1000's of Stores.
www.Shopping.com

Garden **Flowers**
Save on Products in Any Category. Simply Fast Savings.
www.Dealtime.com

See your message here...

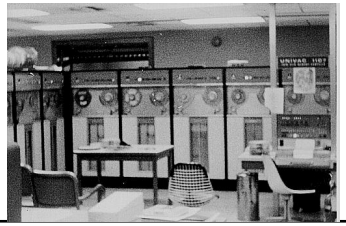
What ads to show and in which order?

Click Through Rate (CTR)



Should CTR discovery be Part of the Sponsored Search Auction?

- The value to the advertiser is the value per click x CTR
- CTR is unknown poses problem of allocation with unknown valuations



- Rank advertisers according to value per click x CTR
- How do they rank new advertisers whose CTR is unknown yet?
- Can they discover CTR while efficiently allocating?

Should CTR discovery be Part of the Sponsored Search Auction?

CTR discovery integrated into the auction

$\Downarrow$

A preference-elicitation mechanism for advertisers and search engines.

CTR discovery by Multi-armed Bandit

- **Multi-armed Bandit:** Classical example of online learning under the explore/exploit tradeoff
  - n advertisers (arms of a bandit) unknown CTR (unknown pay-off as payoff=value per click x CTR)
  - expose k ads (pull k arms) at each time instant to maximize the social welfare (reward) accrued over time
  - Exploit good arms we have learnt so far; explore arms which could be potentially good.
  - There are solutions for Bandit that approximate the optimal expected payoff and achieve this delicate balance.

Multi-armed Bandit with strategic arms and slots with vary quality

- **Multi-armed Bandit:**
  - studied extensively for the non strategic case (e.g. Kleinberg SODA 2006)
  - Even in the context of sponsored search auctions (Pandey, Olston NIPS 2006)
  - for a single good strategic case, a weaker notion of truthfulness with infinite horizon (Bergemann, Valimaki WWW2007)
- **Our goal** is to design a **truthful** mechanism for the strategic case.
  - While allowing for slots with different quality.
- When designing sponsored search auction as an instance of a truthful mechanism for multi-armed bandit
  - by approximating the optimal payoff for the multi-armed bandit
  - the optimal welfare for the auction is approximated.

Multi-armed Bandit with strategic arms and slots with vary quality

- We generalize the Multi-armed Bandit solution in Even-Dar,Manor,Mansour (CLT 2002) due to:
  - Simplicity and optimal sampling complexity

Truthfulness with High Probability

- Since we are sampling CTR by users for finite time horizon
- there is finite probability that sampling is done incorrectly
- and hence will influence our truthfulness.
- Therefore we use the notion of truthfulness with high probability defined in (Archer, Papadimitriou, Talwar and Tardos SODA 2003)

Multi-Armed truthFul bandIt Auction (MAFIA)

A sponsored search auction with different slots quality that is:

- Truthful with high probability
- Has optimal sampling complexity
- And therefore the welfare loss by sampling in bounded.
- The bound is shown to be tight for any sampling keyword algorithm.

The Model

- N risk neutral, utility maximizing advertisers.
- Bid for ad slots based on a single keyword.
- Keyword appear at every time t.
- K slots of advertisements appear in the results.
- Each advertiser i has private value per click v_i (regardless of the slot ad appears at)
- The algorithm runs from time t=1 to t=T(finit time horizon)

Assumptions:

- all advertisers are present in all time periods
- Advertisers have no budget constraints.

Assumptions are relaxed in a companion paper (Gonen,Pavlov 2007)

The Model (cont)

Another assumption:

- The probability for a click in every slot j is
  - monotonically decreasing
  - and is independent of the advertisers.

In order to compare CTR across slots we define normalization probability constants between slots j-1 and j for all $K \geq j > 1$.

Let β_j be the probability for a click in slot j given that i was clicked at the first slot.

The Model (cont)

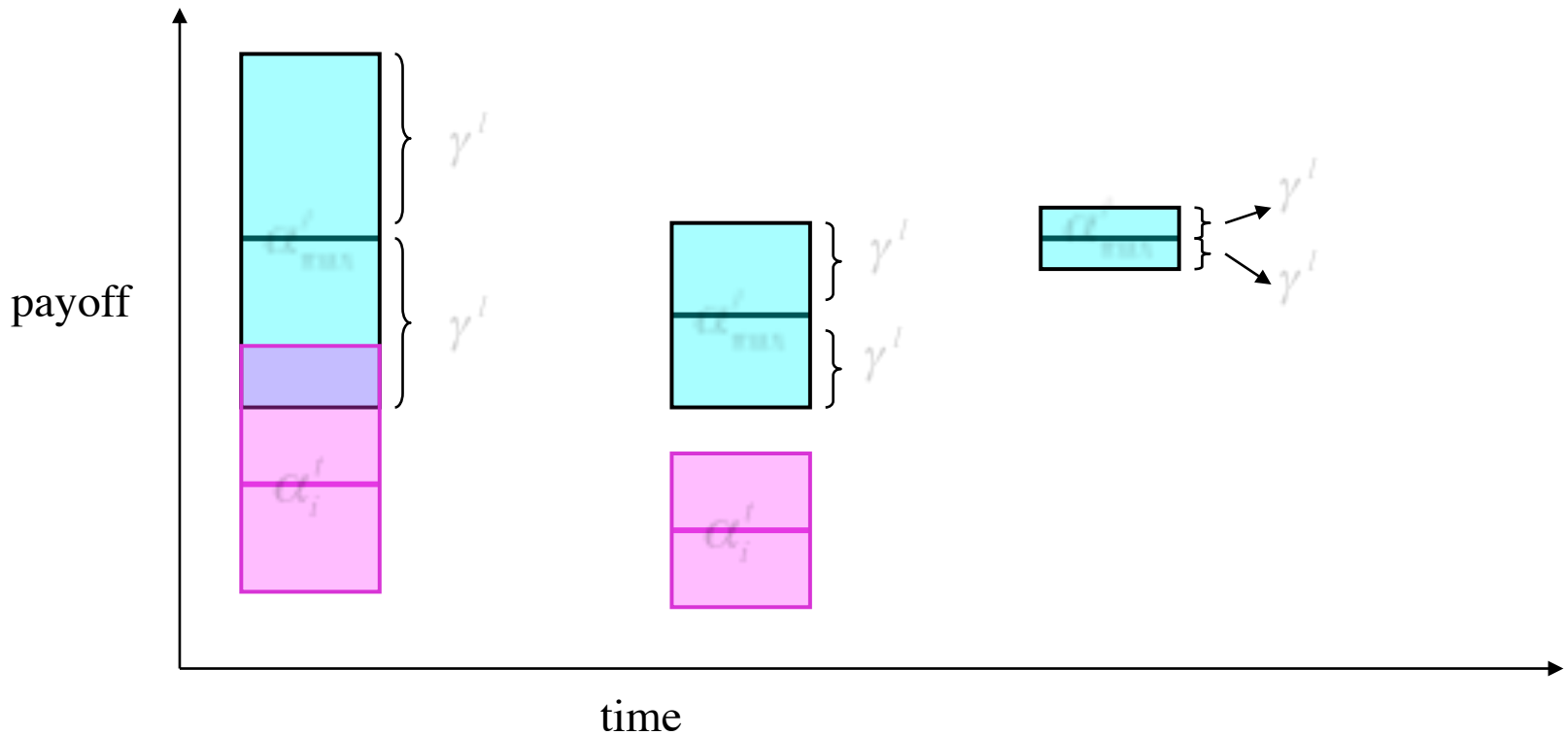
- Each advertiser i has a CTR α_i
  - Which is unknown to i and to the mechanism
- i's observed probability (CTR) at time t is α_i^t.
- i's payoff at time t is $x_i^t = \alpha_i^t \overline{v_i}$
  - Where $\overline{v_i}$ is i's reported value.
- p_i^t is the price advertiser i is charged at time t.
- Advertiser i quasi-linear expected utility for slot j at time t is $\beta_j \alpha_i^t \left(v_i - p_i^t \right)$

Illustrating MAFIA for Single Slot

- The algorithm starts with a set S=N of all advertisers.
- α_i^t of all advertisers i is set to 0.
- For every stage l we have a bound of how accurate α_i^t is
  - Denoted γ^l.
- Once the algorithm learned that advertiser i's estimated payoff is less than the estimated maximal advertiser payoff
  - i is removed from S.
- The process continues until the algorithm is left with the maximal advertiser.

Illustrating MAFIA for Single Slot

- Which advertisers to remove?



Incentive Compatibility for Illustration

- How to motivate strategic advertiser to report $\overline{v}_i = v_i$?

- Given sampled j in S s.t. $\max_{j \in S, j \neq i} x_t^j$

- Advertiser i pays $\dfrac{\alpha_j^t v_j - 2\gamma^l}{\alpha_i^t}$

Generalizing to K vary quality Slots

Three major differences:

1. How to accommodate for CTR in slots of different “quality”
2. How to sample efficiently in a K different “quality” slots environment.
3. How to maintain Incentives in an unequal sampling environment.

Generalizing to K vary quality Slots

Solutions:

1. Normalize CTR to compare all CTR as if were clicked on in first slot.
2. At every stage l order all advertisers i in S according to x_i^t .
   - divide advertisers to K subsets.
   - Align each subset with its corresponding slot.
   - Randomly sample every subset in its slot.
   - Remove advertisers according to the K'th maximal payoff

Generalizing to K vary quality Slots

Solutions:

3. Charge according to ladder price (Aggarwal,Goal,Motwani EC 06):

$$p_i^t = \sum_{f=j}^{K} \frac{\alpha_i^t \beta_{f+1}}{\alpha_i^t \beta_j} \cdot \frac{\alpha_{z_{f+1}}^t}{\alpha_i^t} v_{z_{f+1}}$$

Where z_{f+1} is the maximal among the advertisers that are associated with slot f+1

Note that none of the advertisers taken into account in the ladder price for i is in his subset.

Generalizing to K vary quality Slots

How to allocate slots once only K advertisers are left in S?

In K stages (z=1 to K-1) use confidence parameter γ^{l} to determine which advertiser to remove from the remaining K-z slots while comparing to the K-z max payoff.

Optimal Welfare with Probability $1-\lambda$

- Recall:
  - since we are sampling CTR for finite time horizon
  - there is finite probability that sampling is done incorrectly
  - and hence will influence our truthfulness.
- If we can ensure a high probability $1-\lambda$ of successful sampling we can ensure truthfulness with high probability.
- By tuning γ^l the $1-\lambda$ probability is ensured to be arbitrarily high.

Optimal Welfare with Probability $1-\lambda$

Theorem 1: $\exists\tau$ such that the MAFIA algorithm finds the optimal welfare $\sum_{i\in N}\sum_{t=\tau}^{T}\alpha_i^t v_i$ with probability $1-\lambda$

Proof: main argument

- the observed payoff x_i^t of advertiser i at time t that was not removed is within γ^l of the true payoff x_i.
- As γ^l goes to zero as l increases then after long enough time the advertiser with the best payoff is left.
- For K slots it is shown that the best payoff is achieved in every slot of the K slots and therefore we maximize social welfare for all slots.

Truthfulness with High Probability

$$u_i^1 = \beta_j \alpha_i^t \left(v_i - p_i^t\right)$$

$$u_i^2 = \beta_{j''} \overline{\alpha_i^t} \left(v_i - \overline{p_i^t}\right)$$

$$u_i^3 = \beta_{j'''} \overline{\widehat{\alpha}_i^t} \left(v_i - \overline{p_i^t}\right)$$

$$u_i^4 = \beta_{j'} \widehat{\alpha}_i^t \left(v_i - p_i^t\right)$$

Theorem 2:

given advertiser i and time t, for all $\overline{v_i} \neq v_i$ reported by advertiser i that results in price $\overline{p_i^t}$ it holds that $(1-\lambda)u_i^1 + \lambda u_i^4 \geq (1-\lambda)u_i^2 + \lambda u_i^3$

Truthfulness with High Probability

Proof main arguments:

1. If λ is set to be small enough

$$\lambda \le \min_{i \in N} \left\{ \frac{u_i^1 - u_i^2}{u_i^1 - u_i^2 + \max_{i \in N} \left\{ u_i^3 - u_i^4 \right\}} \right\}$$

   then the probability of the algorithm not finding the optimal welfare results in an arbitrarily low probability of an advertiser being able to gain by lying from a non optimal sampling.

2. So assuming that the algorithm finds the optimal welfare it is shown that no advertiser has incentive to increase or reduce his stated value, meaning

$$\beta_j \alpha_i^t \left(v_i - p_i^t \right) \ge \beta_{j''} \overline{\alpha_i^t} \left(v_i - \overline{p_i^t} \right)$$

Conclusions

This talk presented a truthful multi-armed-bandit mechanism for discovering the valuations of advertisers in a slot-auction game.

The MAFIA mechanism:

- Allows for slots with different quality
- Learns the click-through-rates of the advertisers
- Motivates them to report their true value per click
- Obtains optimal welfare apart from a tightly bounded loss of welfare on the bandit sampling process.
- Bounded sampling complexity

Next Generation

A companion paper (Gonen,Pavlov WINE 2008) relaxes assumptions:

- all advertisers are present in all time periods
- Advertisers have no budget constraints.

By using a new payment scheme it allows for:

- Truthfulness in four parameters; value, arrival time, departure time, and budget.
- Approximates welfare with bounded sampling complexity.

Thank You!