Asynchronous deterministic rendezvous on the line

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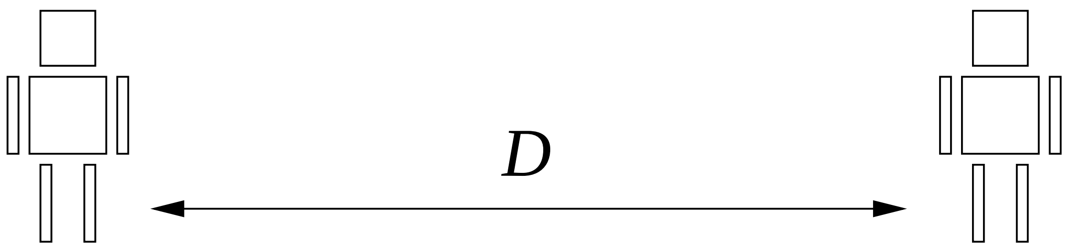
Rendezvous problem

Two robots on the line in distance D want to meet.
None of them knows where the other is.
Have unique labels determining their routes.
Can move on the route at an arbitrary speed which can change in time.



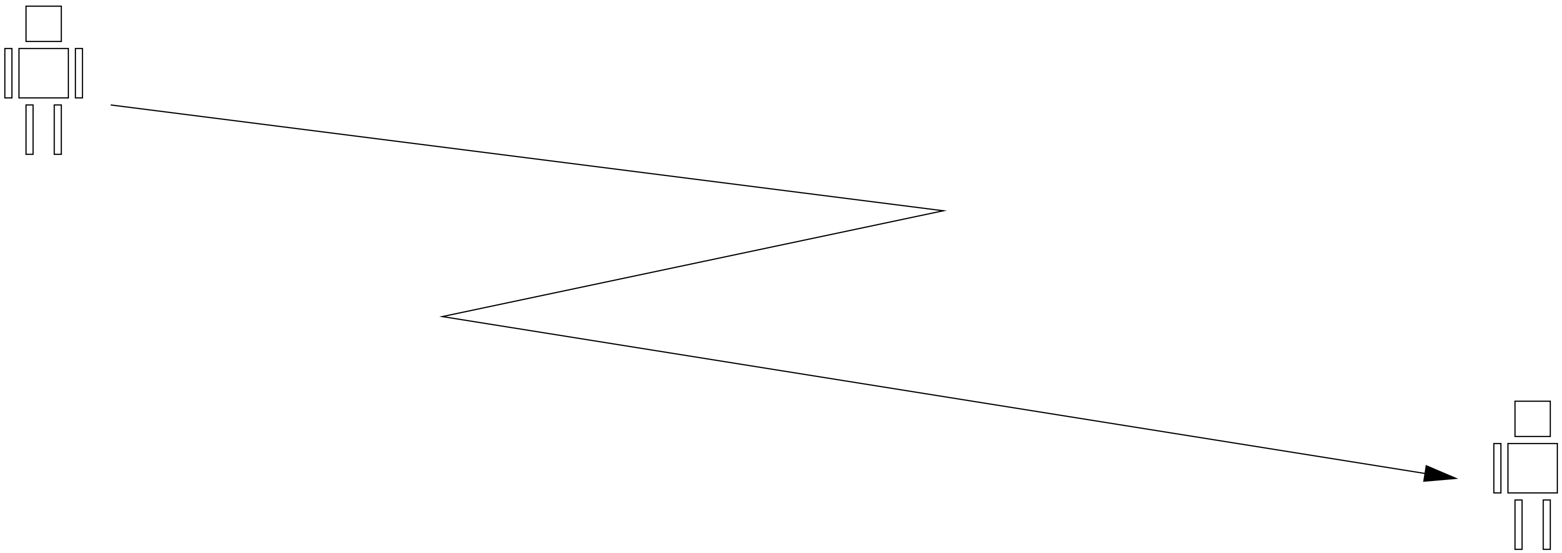
(Special case of the model introduced by De Marco et al.)

Rendezvous complexity

Rendezvous cost is
the maximum total distance covered by both robots.
It depends on D and $|L|$.

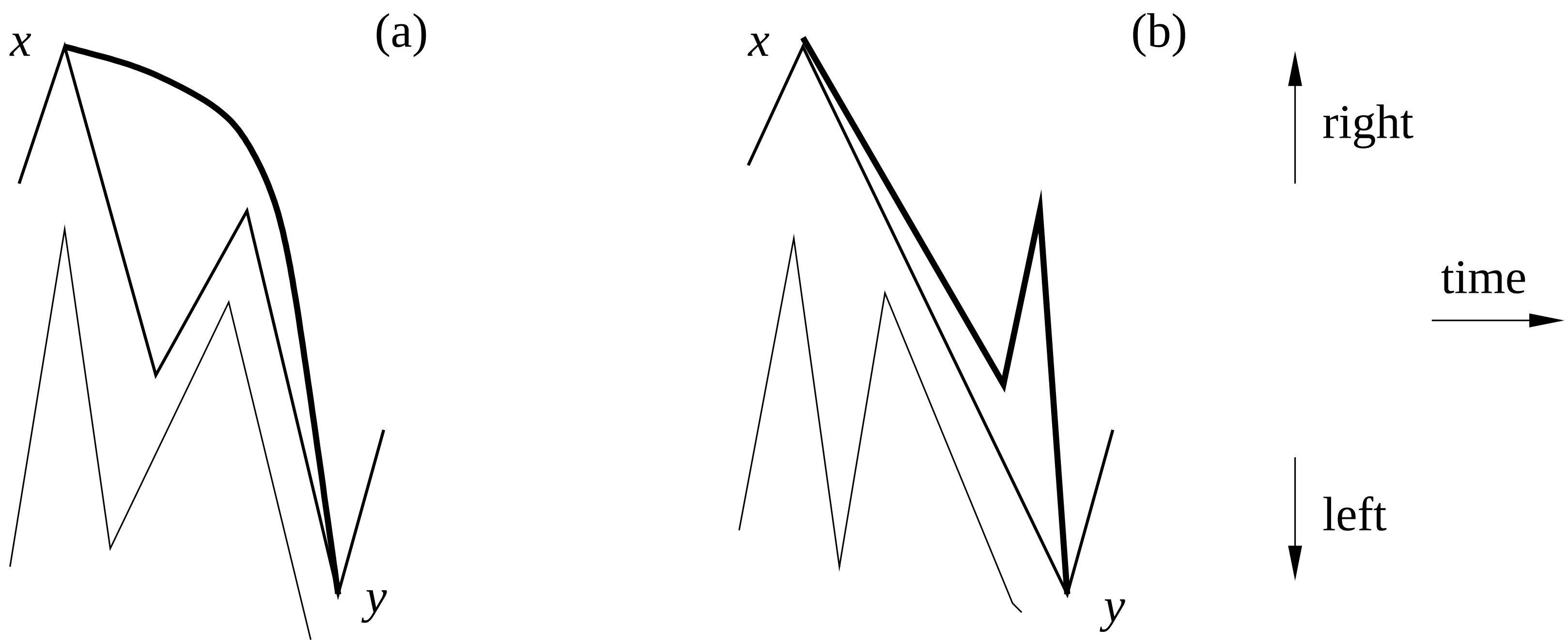
Elimination of lightnings

Labels of robots determine their routes in a rendezvous algorithm.
We show that we can eliminate lightnings from the routes.



Elimination of lightnings

A lightning connecting x and y
can be replaced with segment $[x, y]$ (a)
or vice versa (b)
and two robots all the time do not meet.



Elimination of lightnings

Any route can be expressed as a sequence of its local extrema:

$$x_1, x_2, x_3, x_4, x_5, \ldots$$

If we eliminate lightnings, then the remaining route is either ascending:

$$0 < x_1 < x_3 < x_5 < x_7 < \cdots \text{ and } 0 > x_2 > x_4 > x_6 > x_8 > \cdots,$$

or unimodal:

$$0 < x_1 < x_3 < \cdots < x_k > \cdots > x_n \text{ and}$$

$$0 > x_2 > x_4 > \cdots > x_l < \cdots < x_n \text{ where } |k - l| = 1.$$

Skeleton algorithms

In ith phase the robot gets to the distance s_i from its starting point. If this point if on the LEFT or RIGHT side of the starting point depends on the robot's label L.

Known D

We choose K for any $|L|$ such that $K \sim |L|/2$.
Skeleton:

$$D, 2D, 3D, 4D, \ldots, (K-1)D, KD, KD, (K-1)D, \ldots, 2D, D, 0.$$

We get a skeleton algorithm of cost at most $\sim D|L|^2$.
Best previous algorithm had cost $\sim 8D|L|^2$.

Fixed $|L|$

We choose k such that $k \sim |L|/2$.
Infinite skeleton:

$$b, b^2, b^3, b^4, b^5, b^6, \ldots.$$

Best choice is $b = \sqrt[k]{e}$.
Maximum rendezvous cost $\sim e^2 D|L|^2$

Superposition of skeleton algorithms

We have two skeleton algorithms with ascending skeletons

$$a_1, a_2, a_3, a_4, \ldots \text{ and } b_1, b_2, b_3, b_4, \ldots.$$

We can define $a'_i = a_i + 2 \cdot 2^j$ where $j = \min\{j : 2^j \leq a_j\}$.
Analogously $b'_i = b_i + 3 \cdot 2^j$ where $j = \min\{j : 2^j \leq b_j\}$.
Ascending skeleton of the superposition is the result of merging a'_i and b'_i.

Arbitrary D and $|L|$

We make the superposition of two skeleton algorithms:

1. Algorithm for fixed $|L|$.
2. Algorithm for very different label lengths.

This superposition for $|K| \geq |L|$ has the cost
$O(D\log^2 D + |K|D\log D + D|L|^2 + |K||L|\log|L|)$
Previous best algorithm: $O(D^3 + |K|^3)$

Open problems

1. Is $O(D|L|^2)$ the lower bound on the cost for fixed D?
2. Can we get a fast rendezvous algorithm in the most general setting whose cost depends only on the shorter label L?