The Equational Theory of Weak Complete Simulation Semantics over BCCSP

SOFSEM 2012

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Špindlerův Mlýn, Czech Republic

21 January - 27 January 2012

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   - Definitions and properties
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- We want to extend the previous results on (non)-axiomatizability of process semantics to the weak case.
- In order to do it we translate the unification effort to clarify the concrete ltbt-spectrum, to the weak case.
- In this paper we study the weak complete simulation semantics.
- All the results for the concrete case remain the same, although the existent axiomatizations have to be adequately extended, and we have to develop careful proofs for all the results.

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- Syntax of BCCSP(A_τ)

$$p ::= \; x \mid \mathbf{0} \mid \alpha.p \mid p_1 + p_2 \qquad \alpha \in A_\tau = A \cup \{\tau\}$$

- Operational semantics
  - $p \xrightarrow{\alpha} q \quad (\alpha \in A_\tau)$
  - $p \xRightarrow{\alpha} p \quad (\alpha \in A_\tau)$ (We abstract internal moves)
- Semantics defined by a preorder $\precsim$
  and its kernel $\approx$.
- Equational semantics: Axioms defining $=$ or $\leq$
  Sound axiomatizations.

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- Complete axiomatizations
  - Ground completeness
  - Completeness
  - ω-completeness.
- Axiomatization of the weak bisimulation equivalence.

$$\begin{array}{lrcl} B_1 & x+y & = & y+x \\ B_2 & (x+y)+z & = & x+(y+z) \\ B_3 & x+x & = & x \\ B_4 & x+\mathbf{0} & = & x \end{array} \qquad \begin{array}{lrcl} W_1 & \alpha x & = & \alpha\tau x \\ W_2 & \tau x & = & \tau x + x \\ W_3 & \alpha(\tau x + y) & = & \alpha(\tau x + y) + \alpha y \end{array}$$

$$BW = \{B_1, B_2, B_3, B_4, W_1, W_2, W_3\}$$

- Unification of the equational semantics (concrete case)
  Use of conditional axioms.

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Weak Simulation: definitions and properties

- $\precsim_S$, is the largest relation satisfying that whenever $p \precsim_S q$,

$$\text{If } p \xrightarrow{\alpha} p' \text{ then there exists some } q' \text{ such that } q \stackrel{\alpha}{\Longrightarrow} q' \text{ and } p' \precsim_S q'.$$

- $\approx_S$ is the induced equivalence.
- The preorder $\precsim_S$ is a precongruence over $\mathrm{T}(A_\tau)$.

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Weak Complete Simulation: definitions and properties

- p must terminate ($p \Downarrow$) iff there does not exist any $a \in A$ such that $p \stackrel{a}{\Rightarrow}$.

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Weak Complete Simulation: definitions and properties

- $\tau 0 \precsim_{CS} 0$ but $\tau 0 + a \not\precsim_{CS} a$.
- $\sqsubseteq_{CS}$, is the largest precongruence over $\mathrm{T}(A_\tau)$ included in $\precsim_{CS}$.
- $p \lesssim_{CS} q ::= p \precsim_{CS} q$ and

  Whenever $p \xrightarrow{\tau} p'$ with $p' \Downarrow$, there exists q' such that $q(\xrightarrow{\tau})^+ q'$ and $q' \Downarrow$.

- $\eqsim_{CS}$ is the kernel of $\lesssim_{CS}$.
- $\tau p \lesssim_{CS} p + q$ whenever $p \not\Downarrow$.
- $\lesssim_{CS} = \sqsubseteq_{CS}$.

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Weak Simulation: axiomatization

$$(S) \qquad x \leq x + y$$

$$(\tau e) \qquad \tau x = x$$

Proposition

The set of equations

$$E_{S^{\leq}} = \{B_1, B_2, B_3, B_4, S, \tau e\}$$

is sound and ground-complete for $\mathrm{BCCSP}(A_\tau)$ *modulo* $\precsim_S$.

$W_1 - W_3$ are not needed here, because of the presence of (τe).

Weak Complete Simulation: ground-complete axiomatization

$$(CS_\tau) \qquad (x{\Downarrow} \Leftrightarrow y{\Downarrow}) \Rightarrow x \leq x + y$$

$$(CS_{\tau e}) \qquad \tau(ax + y) = ax + y$$

$$(CS_{\tau e})' \qquad x{\not\Downarrow} \Rightarrow \tau x = x$$

Proposition

The set of equations

$$E^c_{CS_\leq} = BW \cup \{CS_{\tau e}, CS_\tau\},$$

where CS_τ is conditional, is sound and ground-complete for $\mathrm{BCCSP}(A_\tau)$ *modulo* $\lesssim_{CS}$.

Weak Complete Simulation: ground-complete axiomatization

(CS)	$ax \leq ax + y$		
(τN)	$\mathbf{0} \leq \tau\mathbf{0}$	(τg)	$x \leq \tau x$

Proposition

The set of unconditional inequations

$$E_{CS^{\leq}} = BW \cup \{CS_{\tau e}, CS, \tau N\}$$

is sound and ground-complete for $\mathrm{BCCSP}(A_\tau)$ *modulo* $\lesssim_{CS}$.

Weak Simulation: axiomatization of $\approx_S$

$$(SE) \quad a(x+y) = a(x+y) + ay \qquad (a \in A)$$

Proposition

The set of equations

$$E_{S^=} = \{B_1, B_2, B_3, B_4, SE, \tau e\}$$

is sound and ground-complete for $\mathrm{BCCSP}(A_\tau)$ *modulo* $\approx_S$.

Weak Complete Simulation: axiomatization of $\approx_{CS}$

$$(CSE_\tau) \quad (x\Downarrow \Leftrightarrow y\Downarrow) \;\Rightarrow a(x+y) = a(x+y) + ax$$

Proposition

The set of conditional equations

$$E^c_{CS^=} = BW \cup \{CS_{\tau e}, CSE_\tau\}$$

is sound and ground-complete for $\mathrm{BCCSP}(A_\tau)$ *modulo* $\approx_{CS}$.

Weak Complete Simulation: axiomatization of $\eqsim_{CS}$

$$(CSE) \quad a(bx + y + z) = a(bx + y + z) + a(bx + z)$$

Proposition

The set of unconditional equations

$$E_{CS^=} = BW \cup \{CS_{\tau e}, CSE\}$$

is sound and ground-complete for $\mathrm{BCCSP}(A_\tau)$ *modulo* $\eqsim_{CS}$.

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Weak Ready Simulation: definitions and properties

- $I^*(t) = \{a \mid a \in A \text{ and } t \stackrel{a}{\Longrightarrow} t' \text{ for some } t'\}$.
- Weak ready simulation $\precsim_{RS}$, defined by weak simulations whose related pairs $(p, q) \in R$ satisfy $I^*(p) = I^*(q)$.
- $\precsim_{RS}$ is not a precongruence
  $\tau a \precsim_{RS} a$, however, $\tau a + b \not\precsim_{RS} a + b$.
- $\sqsubseteq_{RS}$ is the largest precongruence included in $\precsim_{RS}$.

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Weak Ready Simulation: definitions and properties

Definition

The preorder relation $\lesssim_{RS}$ between processes is defined as follows:
We have $p \lesssim_{RS} q$ iff

- *for any $\alpha \in A_\tau$ such that $p \xrightarrow{\alpha} p'$, there exists some q' such that $q \xRightarrow{\tau} \xrightarrow{\alpha} \xRightarrow{\tau} q'$ with $p' \lesssim_{RS} q'$, and*
- *$I^*(p) = I^*(q)$.*

We denote the kernel of $\lesssim_{RS}$ by $\eqsim_{RS}$.

If $|A| = \infty$ then we have $\sqsubseteq_{RS} = \lesssim_{RS}$.

Axiomatization of $\sqsubseteq_{RS}$ when A is infinite

- (RS_τ) $\quad I^*(x) = I^*(y) \Rightarrow x \leq x + y.$

Proposition

The set of equations

$$E^c_{RS\leq} = BW \cup \{RS_\tau\},$$

in which RS_τ is conditional, is sound and ground-complete for $\lesssim_{RS}$ over the language $\mathrm{BCCSP}(A_\tau)$.

- (RS_τ) can be replaced by the unconditional axioms

$$\begin{array}{ll} (RS) & ax \leq ax + ay \\ (\tau g) & x \leq \tau x \end{array}$$

thus getting $E_{RS\leq}$

Weak Ready Simulation: complete axiomatization

- Completeness:
  If $|A| = \infty$ then $E_{RS\leq}$ is complete.

Theorem

If $|A| < \infty$, no finite set of sound inequations over $\mathrm{BCCSP}(A_\tau)$ modulo $\lesssim^F_{RS}$ can prove all of the sound inequations in the family

$$a^n x \leq a^n \mathbf{0} + \sum_{b \in A} a^n(x+b) \qquad (n \geq 1).$$

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THEOREM

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Complete Simulation: non existence of complete axiomatizations

Proposition

For each $n \geq 0$, the equation

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is sound modulo ready simulation equivalence, and therefore modulo the kernel of $\lesssim_{CS}$.

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$$a^n x + a^n \mathbf{0} + a^n(x + a) = a^n \mathbf{0} + a^n(x + a) \qquad (n \geq 1).$$

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Conclusions and (some) future work

- We have presented the results on (non)-axiomatizability for the complete simulation semantics, thus concluding the study of the weak versions of the most popular simulation semantics.
- Now we are working on the linear-time part of the spectrum, where the definition of the semantics is not still totally clear, because of the subtleties produced by the mixed roles of internal actions (abstraction / non-determinism).

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THANKS!