

Quasiperiodicities in Fibonacci strings

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Periodicity and quasiperiodicity

Periodicity

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Periodicity

Period:

a b a a a b a a a b a a a b a a a b

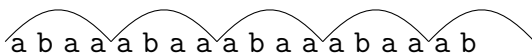
the string is a prefix of some power of the period

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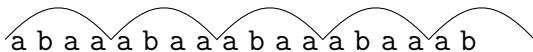
Quasiperiodicity

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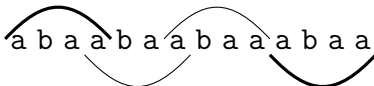
A sequence of characters 'a b a a a b a a a b a a a b a a a b' is shown. Above the characters, a series of connected, downward-pointing arcs connect the first character of each four-character segment to the first character of the next segment, illustrating the periodic nature of the string.

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Quasiperiodicity

Cover:

a b a a b a a b a a a b a a

A sequence of characters 'a b a a b a a b a a a b a a' is shown. Three arcs are drawn above the string, each starting at the first character of a four-character segment and ending at the fourth character of the same segment. The segments are 'a b a a', 'b a a b', and 'a a b a', showing how the cover 'a b a a' overlaps to cover the entire string.

prefix and suffix of the string, such that every letter of the string is within an occurrence of the cover

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Seed:

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a cover of a superstring of the string

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a cover of a superstring of the string

Left seed:seed and prefix of the string

Right seed:seed and suffix of the string

Main related problems

Problem	Worst case	Comments
All Covers	$O(n)$ $O(n)$	Apostolico et al (1991) Moore et al (1994)
Cover array	$O(n)$	Breslauer (1992)
All Seeds	$O(n \log n)$ $O(n)$	Iliopoulos et al (1996) Kociumaka et al (2012)
Seed array	$O(n^2)$	Christou et al (2011)
Left seed array	$O(n)$	Christou et al (2011)
Right seed array	$O(n \log n)$	Christou et al (2011)

Fibonacci strings

Fibonacci strings

A Fibonacci string F_k , $k \in \{0, 1, 2, \dots\}$ is:

$$F_0 = b, \quad F_1 = a, \quad F_n = F_{n-1}F_{n-2} \quad n \in \{2, 3, 4, \dots\}$$

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F_0	b
F_1	a
F_2	ab
F_3	aba
F_4	abaab
F_5	abaababa
F_6	abaababaabaab
F_7	abaababaabaababa

Figure: The first eight Fibonacci strings

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		a	
	a		b
b			a
	a		a
		b	

Figure: The first eight Fibonacci strings

Figure: $C(F_5)$

Interest and results

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- Worst case example for string algorithms
- Bounds on the number of periodicities and quasiperiodicities in strings
- Modern Crystal growth techniques

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Main results on Fibonacci Strings:

- Factors (Chuan et al(2005))
- Borders (Cummings et al(1996))
- Palindromes (Droubay(1995))
- Squares (Iliopoulos et al(1996), Fraenkel et al(1999))
- Cubes (Kubica et al(2009))
- Covers of Circular Fibonacci strings (Iliopoulos et al(1998))

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Our contribution:

- Covers and Seeds of Fibonacci strings and covers of Circular Fibonacci strings**

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Thanks for your attention!