Using Neighborhood Diversity to Solve Hard Problems

Robert Ganian

Faculty of Informatics,
Masaryk University, Brno, Czech Republic.

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Motivation

- Many interesting graph problems are NP-hard.
- Standard approach: use parameterized algorithms.
  - Idea: let the runtime depend on some structural parameter which “captures the complexity” of the graph.
  - Best known parameter: tree-width (low on “tree-like” graphs)
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- However, recent advances have led to problems which cannot be solved by using tree-width.

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Popular choice: \text{Vertex cover}. 
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- Problem: Very restrictive.

The class of graphs with bounded Vertex cover is not very rich.
Is it possible to somehow generalize Vertex cover and still preserve its power as a parameter?
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**Figure:** The **Vertex cover** (6) and **Neighborhood Diversity** (5) of a graph.
The goal was to show that **Neighborhood Diversity** can be used to solve the problems where **Vertex cover** is traditionally used as a parameter.

Since **Neighborhood Diversity** is more general than vertex cover, the obtained parameterized algorithms will be efficient on a larger class of graphs.
Goals and Results

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- However, most parameterized algorithms need to be redesigned from scratch to work on Neighborhood Diversity.

- We provide efficient (FPT) parameterized algorithms for the following problems: p-Vertex-Disjoint Paths, Graph Motif and Precoloring Extension.
The goal was to show that \textit{Neighborhood Diversity} can be used to solve the problems where \textit{Vertex cover} is traditionally used as a parameter.

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Research in this area has lead to the discovery of a more versatile parameter called \textit{Twin-cover} – faster, easier-to-design algorithms (presented at IPEC 2011).