Using Neighborhood Diversity to Solve Hard Problems

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Motivation

- Many interesting graph problems are NP-hard.
- Standard approach: use parameterized algorithms.
  - Idea: let the runtime depend on some structural parameter which "captures the complexity" of the graph.
  - Best known parameter: tree-width (low on "tree-like" graphs)

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- Popular choice: Vertex cover.

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  The class of graphs with bounded Vertex cover is not very rich.
  Is it possible to somehow generalize Vertex cover
  and still preserve its power as a parameter?

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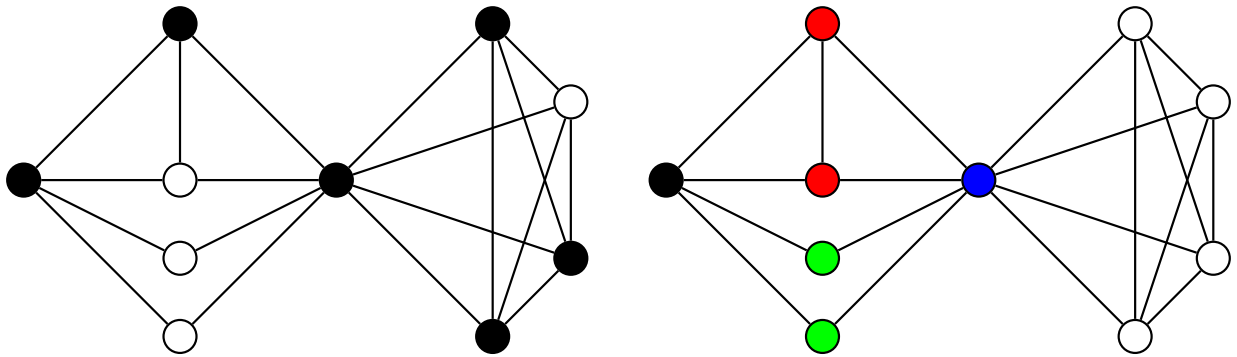

Figure: The Vertex cover (6) and Neighborhood Diversity (5) of a graph

Goals and Results

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- We provide efficient (FPT) parameterized algorithms for the following problems: p-Vertex-Disjoint Paths, Graph Motif and Precoloring Extension.
- Research in this area has lead to the discovery of a more versatile parameter called Twin-cover – faster, easier-to-design algorithms (presented at IPEC 2011).