Logic Characterization of Invisibly Structured Languages: the Case of Floyd Languages

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Outline

1 Background and motivations

2 Floyd's Operator-precedence languages

3 Automata for Operator-precedence languages

4 Logic

5 Applications and Conclusions

Background (1)

Floyd's Operator-precedence languages

- R. Floyd introduced *Operator-precedence (OP) grammars* in 1963, inspired by the structure of arithmetic expressions, and aimed at *deterministic parsing* of programming languages
- Early investigation on the algebraic and closure properties e.g. in [Crespi Reghizzi, Mandrioli and Martin 1978].

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- Early investigation on the algebraic and closure properties e.g. in [Crespi Reghizzi, Mandrioli and Martin 1978].
- Interest on OP languages was "virtually killed" by Knuth's LR algorithm.
- OP parsers are simple and efficient, still sometimes used in practice (e.g. for evaluating arithmetic expressions in gcc)

Background (2)

After 40+ years

- *Visibly Pushdown (VP) languages* [Alur and Madhusudan 2004]
- they exhibit many *closure properties* of regular languages (Boolean, concatenation, Kleene's $*$, ...)
- suitable to model *structured mark-up languages* such as XML; properties enable *model checking*.

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Floyd's OP languages are back

- OP languages actually *include* VP languages and enjoy the *same closure properties*

[Crespi Reghizzi, Mandrioli 2009]

Example: parsing arithmetic expressions

The classical context-free grammar for arithmetic expressions:

$$S \to E$$
$$E \to E + T$$
$$E \to T \times n$$
$$E \to n$$
$$T \to T \times n$$
$$T \to n$$

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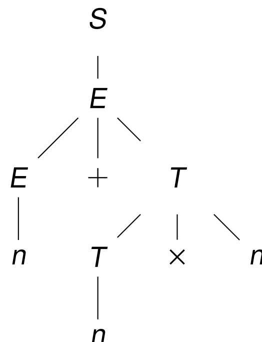
$$n + n \times n$$



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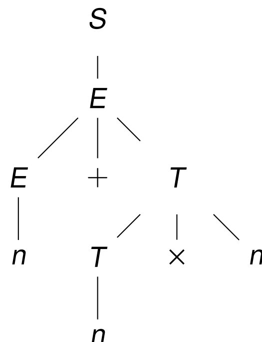

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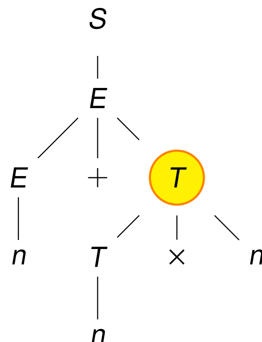

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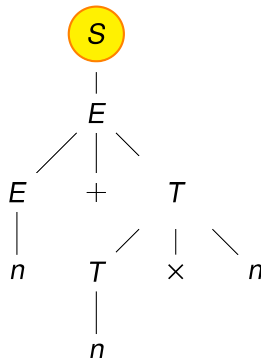

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Floyd's idea

A method for inserting parentheses into an expression is based on *precedence relations* between terminal symbols

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For each pair of symbols we determine the proper precedence and obtain the *precedence table*

Precedence table - example

In a sense, there are no *visible parentheses*, but precedence relations introduce *invisible ones*, e.g.:

Precedence table

$n + n \times n$

	n	$+$	$\times$	$\#$
n		$\gtrdot$	$\gtrdot$	$\gtrdot$
$+$	$\lessdot$	$\gtrdot$	$\lessdot$	$\gtrdot$
$\times$	$\doteq$			$\gtrdot$
$\#$	$\lessdot$	$\lessdot$	$\lessdot$	$\doteq$

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$\#$	$\lessdot$	$\lessdot$	$\lessdot$	$\doteq$

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$$\# \doteq \#$$

Operator grammar

A CF grammar is an *operator grammar* if no right-hand side contains two consecutive nonterminals (and it is not empty).

Floyd's Operator-precedence grammars

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OP grammar

An operator grammar is a *operator-precedence grammar* if there are *no conflicts in its precedence matrix*

i.e. given a pair of symbols, there is *at most one* precedence relation.

Operator-precedence Automata

- OP automata are stack-based
- The stack alphabet consists of pairs *[a q]* where
  a is a symbol from the input alphabet,
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- The stack alphabet consists of pairs $[\,a \;\; q\,]$ where

  a is a symbol from the input alphabet,
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- Variant: the symbol may be marked $[\,a' \;\; q\,]$

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pop from the stack until a *marked symbol c'* is found;

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note: *c' is popped* as well (*look-back*).

e.g. in $n + \overbrace{\lessdot \times n \gtrdot} \#$, $\times$ is marked, $+$ is the look-back and $\#$ the look-ahead.

An example automaton

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- *High priority* means that $\underline{a}$ closes both an a and every pending b after it; e.g.

$$\overbrace{a\ b\ b\ \overbrace{b\ \overbrace{a\underline{a}}\ \underline{b}}\ \underline{a}}$$

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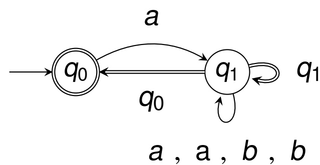
$$\overbrace{a\, b\, b\, \overbrace{b\, \overbrace{a \underline{a}}\, \underline{b}}\, \underline{a}}$$

- also, it is required that the string has only top-level a pairs (e.g. it cannot start with b).

◂ restart ▸▸ skip

Dyck language with parentheses pairs $a, \underline{a}$ (high priority) and $b, \underline{b}$.

	a	$\underline{a}$	b	$\underline{b}$	$\#$
a	$\lessdot$	$\doteq$	$\lessdot$		
$\underline{a}$	$\lessdot$	$\gtrdot$	$\lessdot$	$\gtrdot$	$\gtrdot$
b	$\lessdot$	$\gtrdot$	$\lessdot$	$\doteq$	
$\underline{b}$	$\lessdot$	$\gtrdot$	$\lessdot$	$\gtrdot$	
$\#$	$\lessdot$		$\lessdot$		$\doteq$



$abba\underline{aba}\#$

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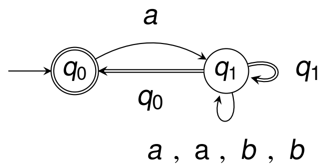
b	$\lessdot$	$\gtrdot$	$\lessdot$	$\doteq$	
$\underline{b}$	$\lessdot$	$\gtrdot$	$\lessdot$	$\gtrdot$	
$\#$	$\lessdot$		$\lessdot$		$\doteq$



$[\# \; q_0]$

$abba\underline{aba}\#$

restart

skip

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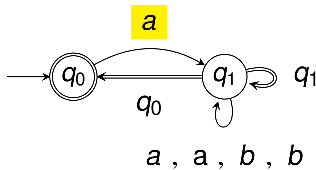
	a	$\underline{a}$	b	$\underline{b}$	$\#$
a	$\lessdot$	$\doteq$	$\lessdot$		
$\underline{a}$	$\lessdot$	$\gtrdot$	$\lessdot$	$\gtrdot$	$\gtrdot$
b	$\lessdot$	$\gtrdot$	$\lessdot$	$\doteq$	
$\underline{b}$	$\lessdot$	$\gtrdot$	$\lessdot$	$\gtrdot$	
$\#$	$\lessdot$		$\lessdot$		$\doteq$



mark

$[\# \; q_0]$

$abba\underline{aba}\#$

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a	$\lessdot$	$\doteq$	$\lessdot$		
$\underline{a}$	$\lessdot$	$\gtrdot$	$\lessdot$	$\gtrdot$	$\gtrdot$
b	$\lessdot$	$\gtrdot$	$\lessdot$	$\doteq$	
$\underline{b}$	$\lessdot$	$\gtrdot$	$\lessdot$	$\gtrdot$	
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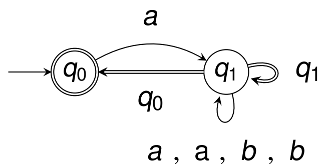

$[\# \; q_0][a' \; q_1]$

$abba\underline{aba}\#$

restart   skip

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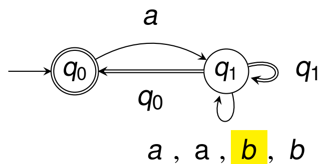
$\underline{b}$	$\lessdot$	$\gtrdot$	$\lessdot$	$\gtrdot$	
$\#$	$\lessdot$		$\lessdot$		$\doteq$



mark

$[\# \; q_0][a' \; q_1]$

$abba\underline{a}\underline{b}\underline{a}\#$

◂ restart ▸▸ skip

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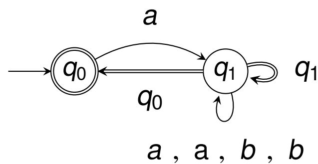
b	$\lessdot$	$\gtrdot$	$\lessdot$	$\doteq$	
$\underline{b}$	$\lessdot$	$\gtrdot$	$\lessdot$	$\gtrdot$	
$\#$	$\lessdot$		$\lessdot$		$\doteq$



$[\# \; q_0][a' \; q_1][b' \; q_1]$

$abba\underline{aba}\#$

An example automaton

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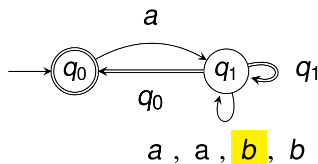
$\underline{b}$	$\lessdot$	$\gtrdot$	$\lessdot$	$\gtrdot$	
$\#$	$\lessdot$		$\lessdot$		$\doteq$



mark

$[\# \; q_0][a' \; q_1][b' \; q_1]$

$ab\,b\,a\,\underline{a}\,b\,\underline{a}\#$

An example automaton

◂ restart ▸▸ skip

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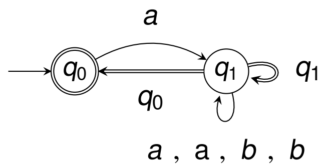
$\underline{a}$	$\lessdot$	$\gtrdot$	$\lessdot$	$\gtrdot$	$\gtrdot$
b	$\lessdot$	$\gtrdot$	$\lessdot$	$\doteq$	
$\underline{b}$	$\lessdot$	$\gtrdot$	$\lessdot$	$\gtrdot$	
$\#$	$\lessdot$		$\lessdot$		$\doteq$



$[\# \; q_0][a' \; q_1][b' \; q_1][b' \; q_1]$

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An example automaton

restart

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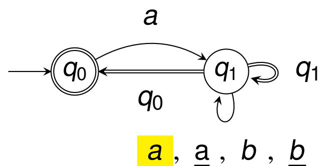

mark

$[\# \; q_0][a' \; q_1][b' \; q_1][b' \; q_1]$

$abba\underline{a}\underline{b}\underline{a}\#$

◂ restart ⏩ skip

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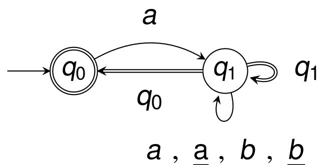
$\#$	$\lessdot$		$\lessdot$		$\doteq$



$[\# \; q_0][a' \; q_1][b' \; q_1][b' \; q_1][a' \; q_1]$ $\quad$ $abba\underline{a}b\underline{a}\#$

restart skip

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$\underline{a}$	$\lessdot$	$\gtrdot$	$\lessdot$	$\gtrdot$	$\gtrdot$
b	$\lessdot$	$\gtrdot$	$\lessdot$	$\doteq$	
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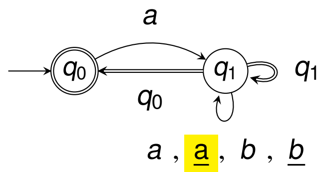

push

$[\# \ q_0][a' \ q_1][b' \ q_1][b' \ q_1][a' \ q_1]$

abba$\underline{aba}$#

An example automaton

◂ restart ▸▸ skip

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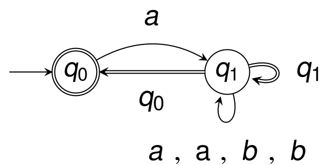
$\#$	$\lessdot$		$\lessdot$		$\doteq$

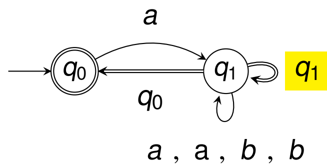


$[\# \; q_0][a' \; q_1][b' \; q_1][b' \; q_1][a' \; q_1][\underline{a} \; q_1]$ $abbaa\underline{a}\underline{b}\underline{a}\#$

restart skip

Dyck language with parentheses pairs $a, \underline{a}$ (high priority) and $b, \underline{b}$.

	a	$\underline{a}$	b	$\underline{b}$	$\#$
a	$\lessdot$	$\doteq$	$\lessdot$		
$\underline{a}$	$\lessdot$	$\gtrdot$	$\lessdot$	$\gtrdot$	$\gtrdot$
b	$\lessdot$	$\gtrdot$	$\lessdot$	$\doteq$	
$\underline{b}$	$\lessdot$	$\gtrdot$	$\lessdot$	$\gtrdot$	
$\#$	$\lessdot$		$\lessdot$		$\doteq$

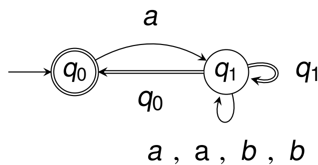


flush $[\# \ q_0][a' \ q_1][b' \ q_1][b' \ q_1][a' \ q_1][\underline{a} \ q_1]$ abbaa$\underline{b}\underline{a}$#

◂ restart ▸▸ skip

Dyck language with parentheses pairs $a, \underline{a}$ (high priority) and $b, \underline{b}$.

	a	$\underline{a}$	b	$\underline{b}$	$\#$
a	$\lessdot$	$\doteq$	$\lessdot$		
$\underline{a}$	$\lessdot$	$\gtrdot$	$\lessdot$	$\gtrdot$	$\gtrdot$
b	$\lessdot$	$\gtrdot$	$\lessdot$	$\doteq$	
$\underline{b}$	$\lessdot$	$\gtrdot$	$\lessdot$	$\gtrdot$	
$\#$	$\lessdot$		$\lessdot$		$\doteq$



$[\# \; q_0][a' \; q_1][b' \; q_1][b' \; q_1]$

$abba\underline{a}\underline{ba}\#$

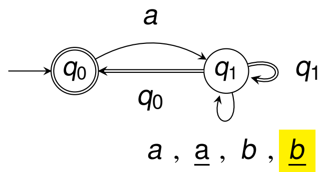
An example automaton

restart

skip

Dyck language with parentheses pairs $a, \underline{a}$ (high priority) and $b, \underline{b}$.

	a	$\underline{a}$	b	$\underline{b}$	$\#$
a	$\lessdot$	$\doteq$	$\lessdot$		
$\underline{a}$	$\lessdot$	$\gtrdot$	$\lessdot$	$\gtrdot$	$\gtrdot$
b	$\lessdot$	$\gtrdot$	$\lessdot$	$\doteq$	
$\underline{b}$	$\lessdot$	$\gtrdot$	$\lessdot$	$\gtrdot$	
$\#$	$\lessdot$		$\lessdot$		$\doteq$



push

$[\# \; q_0][a' \; q_1][b' \; q_1][b' \; q_1]$

$abba\underline{a}\underline{b}\underline{a}\#$

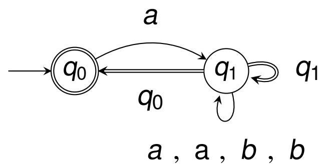
An example automaton

restart

skip

Dyck language with parentheses pairs $a, \underline{a}$ (high priority) and $b, \underline{b}$.

	a	$\underline{a}$	b	$\underline{b}$	$\#$
a	$\lessdot$	$\doteq$	$\lessdot$		
$\underline{a}$	$\lessdot$	$\gtrdot$	$\lessdot$	$\gtrdot$	$\gtrdot$
b	$\lessdot$	$\gtrdot$	$\lessdot$	$\doteq$	
$\underline{b}$	$\lessdot$	$\gtrdot$	$\lessdot$	$\gtrdot$	
$\#$	$\lessdot$		$\lessdot$		$\doteq$

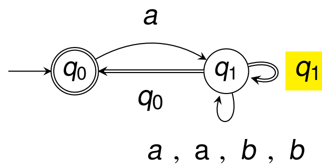


$[\# \; q_0][a' \; q_1][b' \; q_1][b' \; q_1][\underline{b} \; q_1]$ $abba\underline{ab}\underline{a}\#$

◂ restart ▸▸ skip

Dyck language with parentheses pairs $a, \underline{a}$ (high priority) and $b, \underline{b}$.

	a	$\underline{a}$	b	$\underline{b}$	$\#$
a	$\lessdot$	$\doteq$	$\lessdot$		
$\underline{a}$	$\lessdot$	$\gtrdot$	$\lessdot$	$\gtrdot$	$\gtrdot$
b	$\lessdot$	$\gtrdot$	$\lessdot$	$\doteq$	
$\underline{b}$	$\lessdot$	$\gtrdot$	$\lessdot$	$\gtrdot$	
$\#$	$\lessdot$		$\lessdot$		$\doteq$



flush

$[\# \; q_0][a' \; q_1][b' \; q_1][b' \; q_1][\underline{b} \; q_1]$

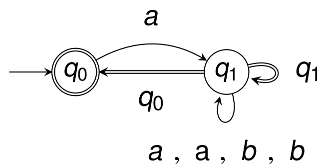
$abbaab\underline{a}\#$

An example automaton

◂ restart ▸▸ skip

Dyck language with parentheses pairs $a, \underline{a}$ (high priority) and $b, \underline{b}$.

	a	$\underline{a}$	b	$\underline{b}$	$\#$
a	$\lessdot$	$\doteq$	$\lessdot$		
$\underline{a}$	$\lessdot$	$\gtrdot$	$\lessdot$	$\gtrdot$	$\gtrdot$
b	$\lessdot$	$\gtrdot$	$\lessdot$	$\doteq$	
$\underline{b}$	$\lessdot$	$\gtrdot$	$\lessdot$	$\gtrdot$	
$\#$	$\lessdot$		$\lessdot$		$\doteq$



$[\# \ q_0][a' \ q_1][b' \ q_1]$

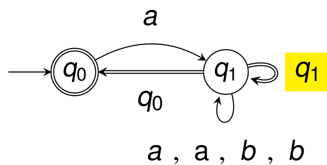
$abba\underline{ab}\underline{a}\#$

An example automaton

◂ restart ▸▸ skip

Dyck language with parentheses pairs $a, \underline{a}$ (high priority) and $b, \underline{b}$.

	a	$\underline{a}$	b	$\underline{b}$	$\#$
a	$\lessdot$	$\doteq$	$\lessdot$		
$\underline{a}$	$\lessdot$	$\gtrdot$	$\lessdot$	$\gtrdot$	$\gtrdot$
b	$\lessdot$	$\gtrdot$	$\lessdot$	$\doteq$	
$\underline{b}$	$\lessdot$	$\gtrdot$	$\lessdot$	$\gtrdot$	
$\#$	$\lessdot$		$\lessdot$		$\doteq$



flush $[\# \; q_0][a' \; q_1][b' \; q_1]$

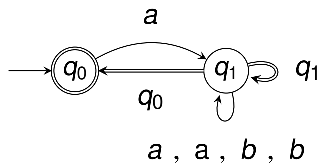
$abbaab\underline{a}\#$

An example automaton

◂ restart ▸▸ skip

Dyck language with parentheses pairs $a, \underline{a}$ (high priority) and $b, \underline{b}$.

	a	$\underline{a}$	b	$\underline{b}$	$\#$
a	$\lessdot$	$\doteq$	$\lessdot$		
$\underline{a}$	$\lessdot$	$\gtrdot$	$\lessdot$	$\gtrdot$	$\gtrdot$
b	$\lessdot$	$\gtrdot$	$\lessdot$	$\doteq$	
$\underline{b}$	$\lessdot$	$\gtrdot$	$\lessdot$	$\gtrdot$	
$\#$	$\lessdot$		$\lessdot$		$\doteq$



$[\# \; q_0][a' \; q_1]$

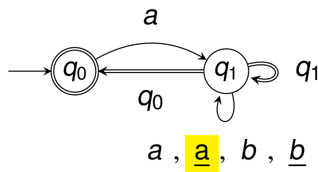
$abba\underline{ab}\underline{a}\#$

restart

skip

Dyck language with parentheses pairs $a, \underline{a}$ (high priority) and $b, \underline{b}$.

	a	$\underline{a}$	b	$\underline{b}$	$\#$
a	$\lessdot$	$\doteq$	$\lessdot$		
$\underline{a}$	$\lessdot$	$\gtrdot$	$\lessdot$	$\gtrdot$	$\gtrdot$
b	$\lessdot$	$\gtrdot$	$\lessdot$	$\doteq$	
$\underline{b}$	$\lessdot$	$\gtrdot$	$\lessdot$	$\gtrdot$	
$\#$	$\lessdot$		$\lessdot$		$\doteq$



push

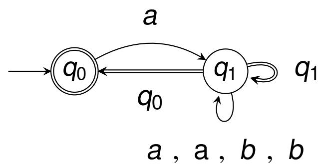
$[\# \ q_0][a' \ q_1]$

$abbaab\underline{a}\#$

◂ restart ▸▸ skip

Dyck language with parentheses pairs $a, \underline{a}$ (high priority) and $b, \underline{b}$.

	a	$\underline{a}$	b	$\underline{b}$	$\#$
a	$\lessdot$	$\doteq$	$\lessdot$		
$\underline{a}$	$\lessdot$	$\gtrdot$	$\lessdot$	$\gtrdot$	$\gtrdot$
b	$\lessdot$	$\gtrdot$	$\lessdot$	$\doteq$	
$\underline{b}$	$\lessdot$	$\gtrdot$	$\lessdot$	$\gtrdot$	
$\#$	$\lessdot$		$\lessdot$		$\doteq$



$[\# \; q_0][a' \; q_1][\underline{a} \; q_1]$

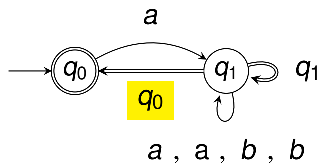
$abbaa\underline{ba}\#$

An example automaton

◂ restart ▸▸ skip

Dyck language with parentheses pairs $a, \underline{a}$ (high priority) and $b, \underline{b}$.

	a	$\underline{a}$	b	$\underline{b}$	$\#$
a	$\lessdot$	$\doteq$	$\lessdot$		
$\underline{a}$	$\lessdot$	$\gtrdot$	$\lessdot$	$\gtrdot$	$\gtrdot$
b	$\lessdot$	$\gtrdot$	$\lessdot$	$\doteq$	
$\underline{b}$	$\lessdot$	$\gtrdot$	$\lessdot$	$\gtrdot$	
$\#$	$\lessdot$		$\lessdot$		$\doteq$



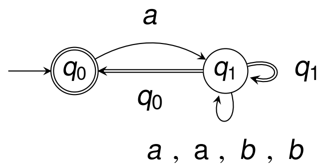
flush $[\# \, q_0][a' \, q_1][\underline{a} \, q_1]$

$abba\underline{aba}\#$

◂ restart ▸▸ skip

Dyck language with parentheses pairs $a, \underline{a}$ (high priority) and $b, \underline{b}$.

	a	$\underline{a}$	b	$\underline{b}$	$\#$
a	$\lessdot$	$\doteq$	$\lessdot$		
$\underline{a}$	$\lessdot$	$\gtrdot$	$\lessdot$	$\gtrdot$	$\gtrdot$
b	$\lessdot$	$\gtrdot$	$\lessdot$	$\doteq$	
$\underline{b}$	$\lessdot$	$\gtrdot$	$\lessdot$	$\gtrdot$	
$\#$	$\lessdot$		$\lessdot$		$\doteq$



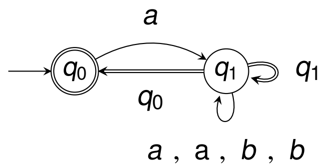
$[\# \; q_0]$

$abbaa\underline{ba}\#$

◂ restart ▸▸ skip

Dyck language with parentheses pairs $a, \underline{a}$ (high priority) and $b, \underline{b}$.

	a	$\underline{a}$	b	$\underline{b}$	$\#$
a	$\lessdot$	$\doteq$	$\lessdot$		
$\underline{a}$	$\lessdot$	$\gtrdot$	$\lessdot$	$\gtrdot$	$\gtrdot$
b	$\lessdot$	$\gtrdot$	$\lessdot$	$\doteq$	
$\underline{b}$	$\lessdot$	$\gtrdot$	$\lessdot$	$\gtrdot$	
$\#$	$\lessdot$		$\lessdot$		$\doteq$



accept $[\# \; q_0]$

$abbaaba\#$

Equivalence theorems

- Deterministic OP automata are equivalent to nondeterministic ones.
- OP grammars and automata have the *same expressive power*.

[Lonati, Mandrioli, Pradella 2011]

A logic for operator-precedence languages

- The $\mathrm{MSO}_{\Sigma,M}$ (*monadic second-order logic*) is interpreted over (Σ, M) *strings*, and it is defined by the following syntax
- where $a \in \Sigma$, x, y are first-order variables (representing *positions* in the string) and X is a set variable.
- $\varphi :=$ $a(x) \mid x \in X \mid x \leq y \mid x = y + 1 \mid \neg\varphi \mid \varphi \vee \varphi \mid \exists x.\varphi \mid \exists X.\varphi \mid$ $x \curvearrowright y$ *matching relation*

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- Intuitively, $x \curvearrowright y$ holds iff positions x and y are to become *"adjacent" during parsing*
- i.e. the symbol at x is used as *look-back*, and the one at y as *look-ahead* in a *flush* move.

Logic-Automata equivalence

- The construction from logic to automata is quite standard
- From automata to logic, it is not:

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  - Finite-state and VP automata are *real-time* machines;
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- From automata to logic, it is not:
  - Finite-state and VP automata are *real-time* machines;
  - OP automata are not, so it is not easy to represent their runtime configuration in logic;
  - Indeed $\curvearrowright$ represents a relation that is *not* one-to-one (the structure can be *invisible*).
- In parenthesis languages (like VPL) the structure is *visible*; the analogous of $\curvearrowright$ in VPL matches parentheses (*one-to-one*).

Applications

Parallel and incremental parsing

OP languages are *locally parsable*: i.e. we can parse substrings without modifying the outcome when considering larger strings (*PAPAGENO*)

[Barenghi, Crespi Reghizzi, Mandrioli, Pradella 2012; 2013]

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Model checking

Closure properties enable *infinite-state model checking*.

OP languages can model some characteristics exhibited by real-world systems, as *transactions*, *interrupts* and *hierarchical management* (e.g. in operating systems, distributed databases, privilege-based accesses, versioning systems).

Conclusions

OP automata and logic

We defined OP automata and a MSO logic that perfectly matches the generative power of OP grammars

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Operator-precedence languages

OP languages have *almost the full generality of deterministic context-free* but *enjoy all the nice properties* of regular and VP languages.