On the State Complexity of Ultrametric Finite Automata

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p-adic absolute values

- Any rational non-zero number can be expressed as $\alpha = \pm 2^{\alpha_2} * 3^{\alpha_3} * 5^{\alpha_5} * 7^{\alpha_7} * 11^{\alpha_{11}} * \cdots$, where all α_i are integers
- From this, the p-adic absolute value (also called p-norm) of α for a prime p is defined as $\|\alpha\|_p = \begin{cases} 0, if\ \alpha = 0 \\ p^{-\alpha_p}\ otherwise \end{cases}$

Ultrametric finite automata (UFA)

- Generalization of probabilistic finite automata
- Instead of transition probabilities, use transition *amplitudes* – p-adic numbers
- When determining whether or not to accept a word, the p-norms of all accepting state amplitudes are calculated and added together
- If the sum is at least (or at most, depending on the automaton) a given treshold, the automaton accepts the word

Language $L_{k,m}$

- Let $w = (w_1, w_2, \dots, w_m) \in \{0,1, \dots k-1\}^m$
- Consider two operations on w:
  a) Cyclic shift: $f_a(w) = (w_m, w_1, w_2, \dots, w_{m-1})$
  b) Increasing first element: $f_b(w) = \left((w_1 + 1)\ mod\ k, w_2, \dots, w_m\right)$
- Let $x \in \{a, b\}^n$. Define $f_{x_1 x_2 \dots x_n}(w) = f_{x_n}\left(\cdots f_{x_2}\left(f_{x_1}(w)\right)\cdots\right)$
- $L_{k,m} = \{x \in \{a, b\}^* | f_x(0^m) = 0^m\}$

Recognizing $L_{k,m}$

- Deterministic finite automaton – at least k^m states are necessary
- For any prime p, there is a p-ultrametric finite automaton that can recognize $L_{k,m}$ with $k * m$ states
- For every prime p, if p > m, there is a p-UFA that can recognize $L_{p,m}$ with $m + 1$ states
- Visit us at the poster session to see proofs of the above ☺