On Contact Graphs with Cubes and Proportional Boxes

Jawaherul Alam [1] Michael Kaufmann [2]
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[1]University of California - Irvine

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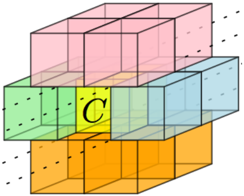
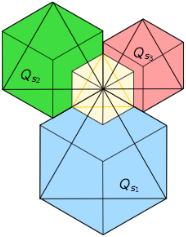
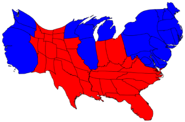


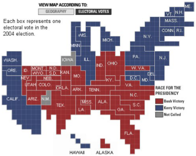


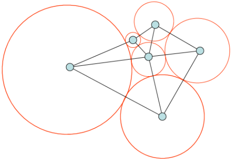
Contact Representation of Graphs

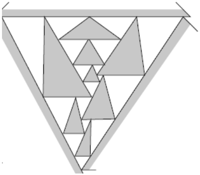
Many different types of contact

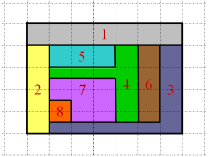
- circles, triangles, rectangles, boxes, ...
- point contact vs. side (proper) contact
- unweighted vs. weighted (proportional)
- 2D, 3D
- ...

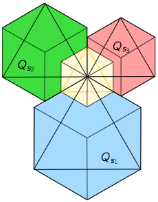




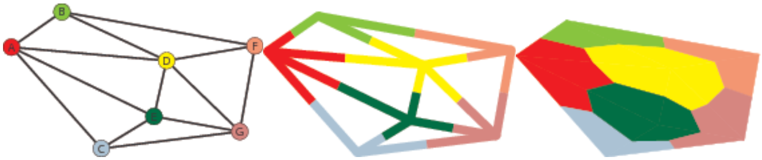






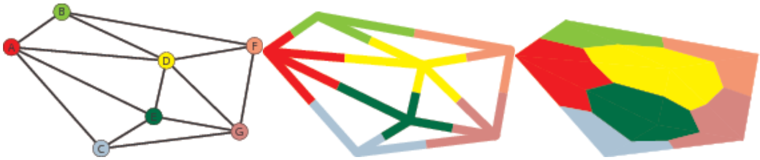


Contact Representations



- vertices: polygons
- edges: non-trivial borders

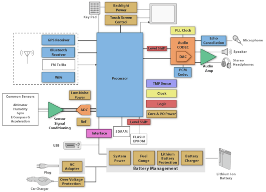
Proportional Contact Representations

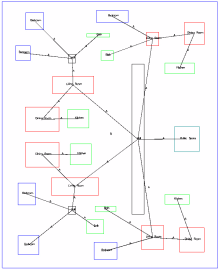


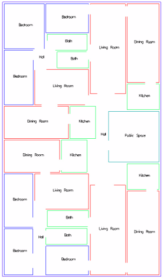
- vertices: polygons
- edges: non-trivial borders
- *vertex weight $\Rightarrow$ area of polygon, or volume of polytope*

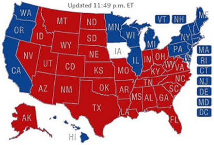
Motivation

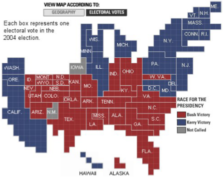
- VLSI Layout
- floorplanning
- data visualization





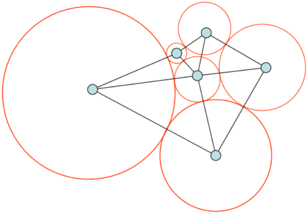


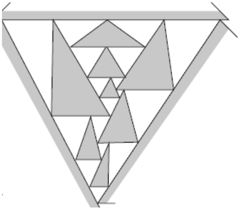


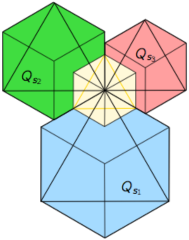


Related Work

- contact with circles [Koebe, 1936]
- contact with triangles [de Fraysseix *et al.*, 1994]
- contact with 3D cubes [Felsner and Francis, 2011].

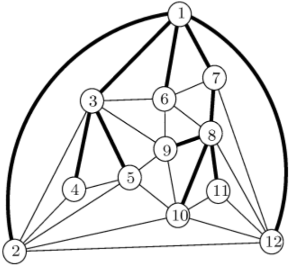


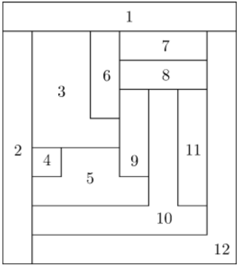




Related Work cont.

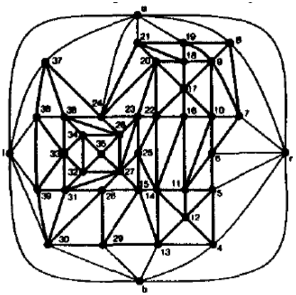
Rectilinear Duals: Eight-sided rectilinear polygons are always sufficient and sometimes necessary for maximal planar graphs [Yeap and Sarrafzadeh 1993, He 1999, Liao *et al.*, 2003].

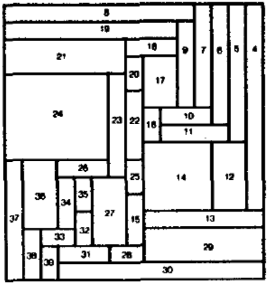




Related Work cont.

Rectilinear Duals: Rectangles are sufficient for 4-connected maximal planar graphs [Ungar 1953, Kozminski and Kinnen 1985, Kant and He, 1997]





Related Work cont.

Connections...

- The edges of any maximal planar graph can be partitioned into 3 edge-disjoint spanning trees [Nash 1961, Tutte 1961]

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Related Work cont.

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- The edges of any maximal planar graph can be partitioned into 3 edge-disjoint spanning trees [Nash 1961, Tutte 1961]
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- Canonical order defined and used for straight-line drawing [de Fraysseix, Pach and Pollack 1990]

Related Work cont.

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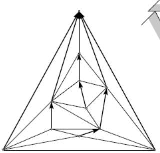
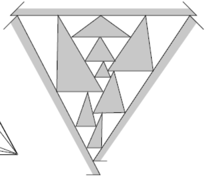
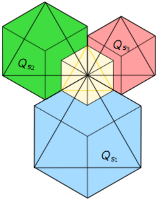
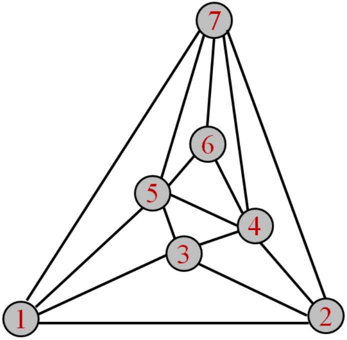
- The edges of any maximal planar graph can be partitioned into 3 edge-disjoint spanning trees [Nash 1961, Tutte 1961]
- Schnyder realizer was defined and used for straight-line drawing [Schnyder 1990]
- Canonical order defined and used for straight-line drawing [de Fraysseix, Pach and Pollack 1990]
- Relations between canonical order, Schnyder realizer used to prove various results [de Fraysseix, Kant, He, Felsner, Fusy, Ueckerdt,...]

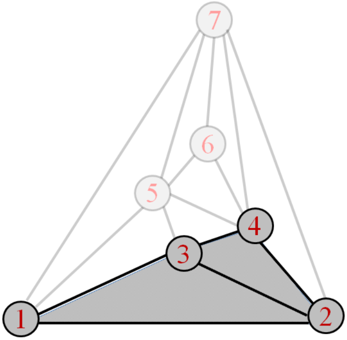






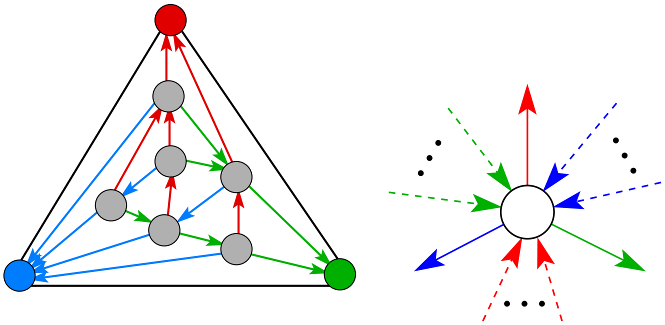
Canonical Order





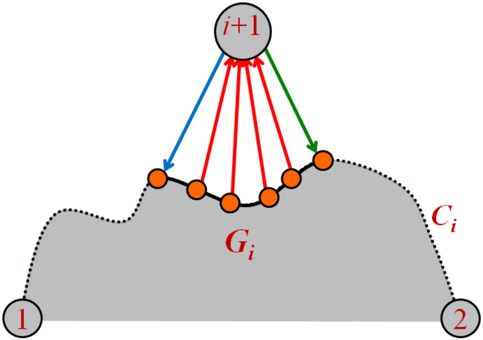
- Ordering of the vertices
- G_i (induced on vertices $1, \ldots, i$) is biconnected
- $i + 1$ is on outerface of G_{i+1}
- $i + 1$ has 2 or more consecutive neighbors on C_i

Schnyder Realizer



- partition of the internal edges into three spanning trees
- every vertex has out-degree exactly one in T_1, T_2 and T_3
- vertex rule: ccw order of edges: entering T_1, leaving T_2, entering T_3, leaving T_1, entering T_2, leaving T_3

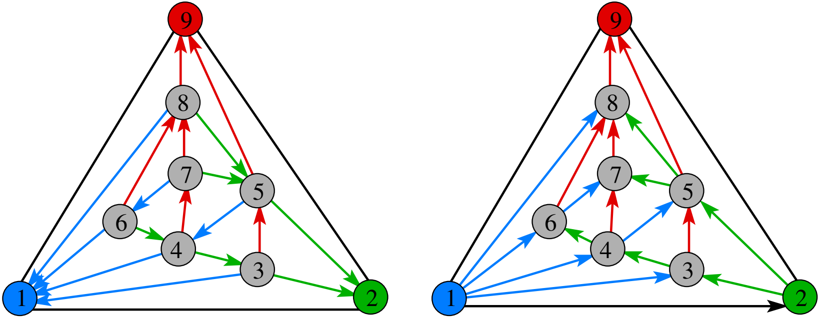
From Canonical Order to Schnyder Realizer



When a new vertex is inserted in the canonical order:

- leftmost edge is outgoing blue
- rightmost edge is outgoing green
- remaining (0 or more edges) incoming red
- (it gets its outgoing red when it is “closed off”)

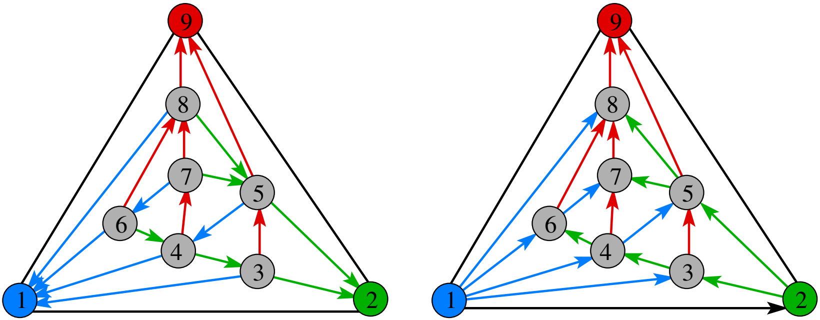
From Schnyder Realizer to Canonical Order



Two easy ways:

- counterclockwise preorder traversal of the blue tree

From Schnyder Realizer to Canonical Order

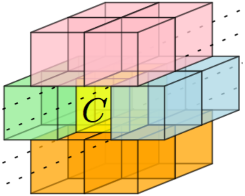


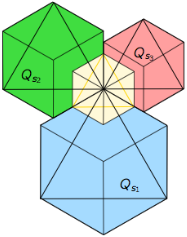
Two easy ways:

- counterclockwise preorder traversal of the blue tree
- topological order of $T_1 \cup T_2^{-1} \cup T_3^{-1}$

Contacts in 3D

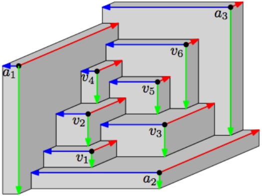
- A planar graph has a representation using axis-parallel boxes in 3D, where two boxes have a non-empty intersection iff their corresponding vertices are adjacent. It holds with contacts rather than intersections.
  [Thomassen 1986]
- A planar graph has a contact representation (improper) with axis-parallel cubes in 3D, where two boxes touch iff their corresponding vertices are adjacent.
  [Felsner and Francis 2011]

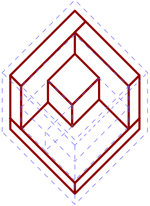




Contacts in 3D

We study proper contact representations: in 3D touching objects have non-trivial-area face overlap





[Bremner, Kobourov et al., GD'12]

- Deciding unit cube proper contact is NP-Complete
- Two new proofs of Thomassen's proper box contact

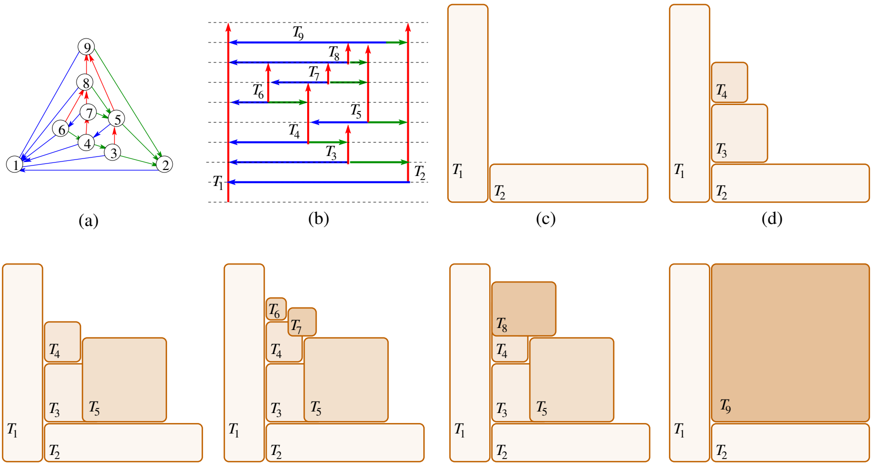
[Alam, Evans, Kobourov et al., WADS'15]

- 3-connected planar: proper primal-dual box-contact repr.
- 1-planar prime graphs: proper box-contact repr.

Box Representation via FPP

Theorem

(Thomassen) Planar graphs have touching boxes contact representation.

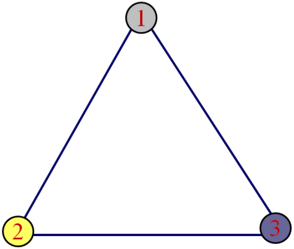


Cube Representation for Planar 3-Trees

Theorem

Every planar 3-tree has a proper contact representation by cubes.

Recall planar 3-trees: either a 3-cycle or a planar graph G with vertex v, s.t., $deg(v) = 3$ and $G - v$ is a planar 3-tree



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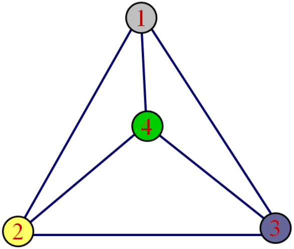

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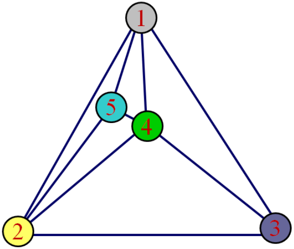

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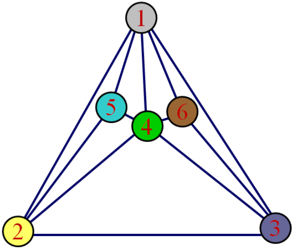

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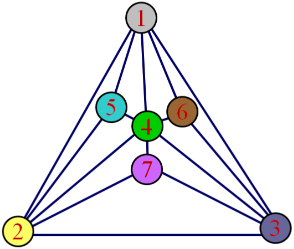

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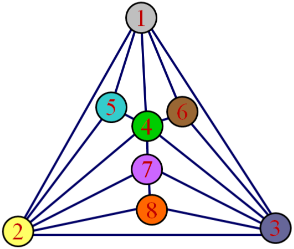

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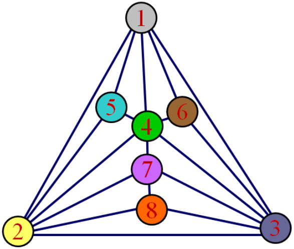
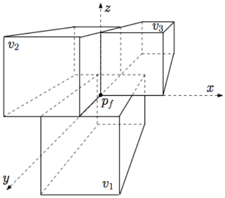
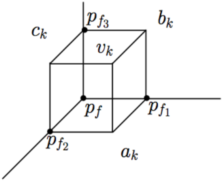






Proportional Box Representation

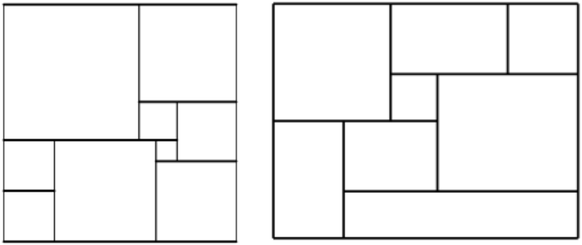
Theorem

Every internally triangulated 4-connected planar graph has a proper proportional contact representation with boxes.

Proof.

- 4-connected plane graph $\rightarrow$ rectangulation
- "grow" boxes up to get desired volumes

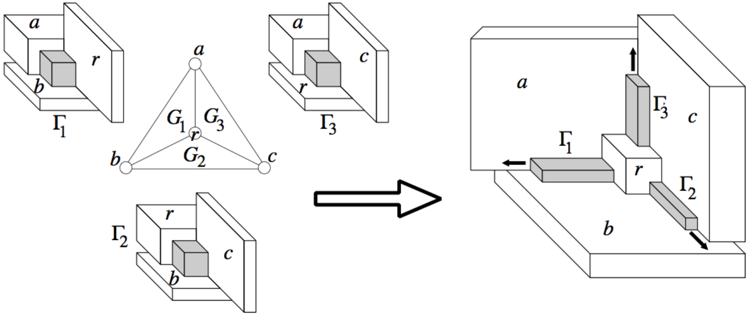
$\square$



Proportional Box Representation for Planar 3-Trees

Theorem

Let $G = (V, E)$ be a plane 3-tree with a weight function w. Then a **proportional box-contact representation** *of G can be computed in linear time.*



Proportional Box Representation for Planar 3-Trees

Theorem

Let $G = (V, E)$ be a plane 3-tree with a weight function w. Then a **proportional box-contact representation** *of G can be computed in linear time.*

Proof.

- compute representative tree T for G (aka 4-block tree)
- U_v: the set of the descendants of v in T_G including v.
- *predecessors* of v are $N_G(v)$ that are not in U_v
- scale weights so that $w(v) \geq 1 \ \forall v \in G$
- let v_1, v_2 and v_3 be the three children of v in T_G
- define $W(v) = \Pi_{i=1}^{3}[W(v_i) + \sqrt[3]{w(v)}]$
- compute bottom up

□

What other graphs have such representations?

Theorem

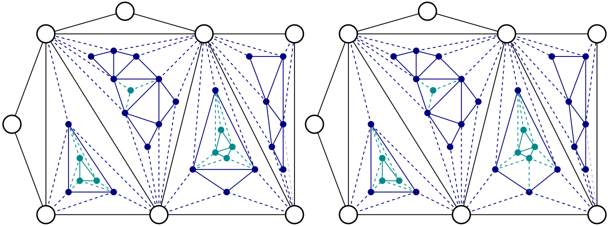
Any nested maximal outerplanar graph has a proper contact representation with cubes.

Theorem

Let $G = (V, E)$ be a nested outerplanar graph with a weight function w. Then a **proportional box-contact representation** *of G can be computed in linear time.*

Nested Outerplanar Graphs

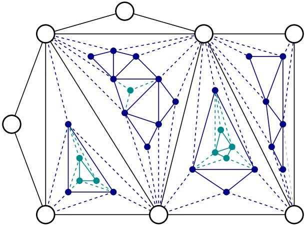
A nested outerplanar graph, is kind of like a *k*-outerplanar graph (where, a 1-outerplanar graph is the standard outerplanar).



Definition

A nested outerplanar graph is either an outerplanar graph or a planar graph G, where each component induced by the internal vertices is another nested outerplanar graph, with exactly three neighbors in the outerface of G.

Nested Maximal Outerplanar Graphs



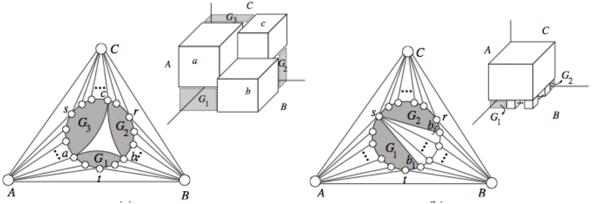
Definition

A nested maximal outerplanar graph is a subclass of nested outerplanar graphs, that is either a maximal outerplanar graph, or a maximal planar graph in which the vertices on the outerface induce a maximal outerplanar graph and each component induced by internal vertices is another nested maximal outerplanar graph.

3D Representations

Theorem

Any nested maximal outerplanar graph has a proper contact representation with cubes.

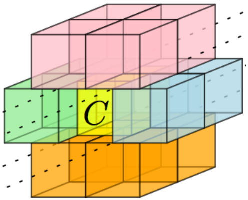


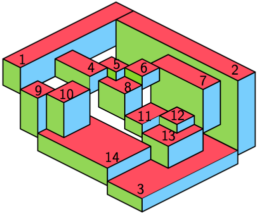
Theorem

Let $G = (V, E)$ be a nested outerplanar graph with a weight function w. Then a **proportional box-contact representation** *of G can be computed in linear time.*

Open Problems

- Contact Representation in 3D: do all planar graphs have proper cube contact representation?
- Proportional Contact Representation in 3D: do all planar graphs have proper proportional box contact representation?





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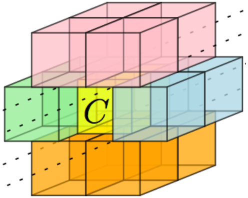
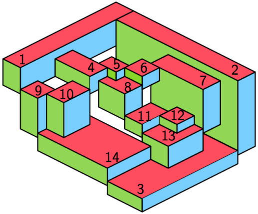
- Proportional Contact Representation in 3D: do all planar graphs have proper proportional box contact representation?





Going back to Prague/Olomouc on Wednesday?

I need to be in Prague by 13:00, or in Olomouc by 17:30.
Help! email: *kobourov@cs.arizona.edu*