LQD is 1.5-competitive for 3-port shared-memory switches

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A shared-memory switch and the LQD algorithm

- A shared-memory switch consists of a buffer of size M and an equal number N of input ports and output ports.
- Assuming that time is slotted, packets arrive at any time step and should be either accepted by the buffer of the switch or be irreversibly rejected by the switch.
- Each arriving packet has a single destination output port.
- According to the LQD online algorithm, each packet is accepted if the buffer is not full. If the buffer is full, then a packet is preempted from the longest queue in the buffer and the arriving packet is accepted.
- It has been shown [Hahne et al., SPAA 2001] that LQD is 2-competitive and at least $\sqrt{2}$-competitive.
- For the case of 2-port switches, LQD has been shown to be exactly 4/3-competitive [Kobayashi et al., SPAA 2007]
- We show that LQD is 1.5-competitive for 3-port shared-memory switches.

Analysis

- Most of our results hold for shared-memory switches equipped with any number of output ports $N \geq 2$ and are not restricted to 3-port switches.
- A queue is said to be *active* in the LQD (OPT) buffer if its length at the current time step is at least 1 in this buffer.
- The LQD buffer is said to *overflow* at a time step, if at least one packet destined to any queue is rejected or preempted by this buffer.
- There exists an optimal offline algorithm (OPT) which never keeps inactive a queue that is active in the LQD buffer at the same time step.
- We shall follow the usual approach of assigning connections between packets transmitted by OPT and packets transmitted by LQD, proving that for each additional packet that OPT transmits, LQD has already transmitted at least two packets.

Analysis

- We, subsequently, derive some technical results regarding the length that any queue that overflows in the LQD may attain in the OPT buffer, at the same time step.
- As an example, if two queues simultaneously overflow in the LQD buffer, their respective length is upper-bounded by M/2.
- The analysis continues, by deriving upper-bounds on the length that any queue that overflows in the LQD buffer may attain in the OPT buffer at the same time step.
- After establishing these results, we start assigning connections between OPT packets and LQD packets.
- For this, we distinguish between cases.
- For example, we distinguish between the cases that a queue in the OPT buffer has at most twice the length of the same queue in the LQD buffer or more.

Conclusions

- We show that LQD is 1.5-competitive for 3-port shared-memory switches.
- It is one of the important open problems in the area of buffering problems, to improve upon the upper-bound of 2 for the LQD competitive ratio (for arbitrary N).
- Some of our results could be used to improve on the upper bound of 2, since they hold for shared-memory switches equipped with an arbitrary number of output ports N.